Inflation Inattention on the Production Network: Firm-Level Evidence and Macro Implications

Rafael Gonçalves Ina Hajdini

September 2026

Abstract

Relying on a micro-level dataset of US firms' inflation expectations, we document that firms in industries with larger Domar weights and more flexible prices have more accurate inflation forecasts. To explain these patterns, we develop a rational inattention model of price-setting firms within a production network. In equilibrium, firms' inflation forecast accuracy depends on their attention to marginal costs, the comovement between their marginal cost and aggregate inflation, and the change in the marginal cost. When calibrated to US input-output data, the model replicates the cross-sectoral relationship between forecast accuracy, Domar weights, and price flexibility. Quantitatively, we find that production networks endogenously increase nominal rigidity. This channel accounts for about 30 percent of the networks' attenuation of the inflation response to monetary shocks, deepening the standard flattening of the Phillips curve from input-output linkages.

Keywords: inflation expectations, rational inattention, production networks, inflation dynamics, monetary policy.
JEL codes: E31, E32, E71.

Gonçalves: Princeton University, rgoncalves@princeton.edu. Hajdini: Federal Reserve Bank of Cleveland, Ina.Hajdini@clev.frb.org. We thank Zhen Huo, Damjan Pfajfar, Karthik Sastry, Gianluca Violante, participants at PUC-Rio, Paris Conference on the Macroeconomics of Expectations, Midwest Macroeconomics Meeting, CEBRA Annual Meeting, SED Annual Meeting, and CESifo Area Conference on Macro with Micro Data conference for helpful comments. The views expressed here are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Cleveland or the Federal Reserve System.

1. Introduction

Modern macroeconomic theory attributes a crucial role to firms' inflation expectations in shaping the dynamics of both nominal and real variables. However, limited data on firms' expectations have hindered our understanding of the joint distribution of firms' price decisions, their beliefs, and macroeconomic outcomes. In this context, the full-information rational expectations (FIRE) framework has served as the paradigm for modeling expectations and their influence on macroeconomic dynamics in recent decades.

A growing empirical literature has challenged this paradigm, documenting substantial heterogeneity and inaccuracy in firms' inflation expectations (Coibion, Gorodnichenko, and Kumar 2018; Candia, Coibion, and Gorodnichenko 2024, among others). Notably, firms' "inattention" to inflation appears related to industry-specific characteristics, such as realized input price changes (Andrade et al. 2022) and the number of competitors (Afrouzi 2024). However, whether other relevant industry attributes, such as an industry's importance in the production network or its degree of price rigidity, shape firms' expectations remains underexplored.

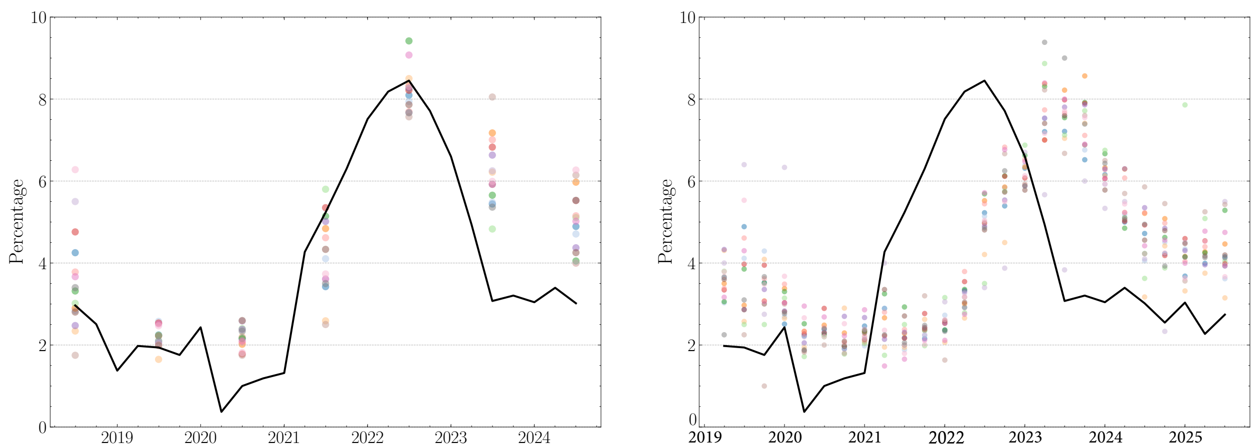
This paper investigates the causes and macroeconomic consequences of heterogeneity in firms' inflation expectations, with a focus on the role of industry differences. On the empirical front, we combine a novel and comprehensive dataset on US firms' inflation expectations with industry-level data on production structure and pricing behavior. We document that the accuracy of firms' inflation forecasts is systematically associated with industry attributes: firms in industries with larger Domar weights (sales-to-GDP ratios) and more flexible prices have significantly more accurate forecasts. We further document that firms' inflation expectations respond to industry-specific marginal cost changes, consistent with firms using their own cost conditions as a signal about aggregate inflation.

Motivated by these empirical patterns, we develop a rational inattention model of price-setting firms operating within a production network. We first analyze a static special case to derive analytical results. The model implies that aggregate inflation nowcast accuracy is increasing in both the firm's attention level and the R-squared of a regression of aggregate inflation on industry-level marginal cost changes (which we show to be related to that industry's Domar weight).

We then study the implications of rational inattention for the transmission of monetary shocks. We show that stronger input-output linkages affect the Phillips curve slope via two channels. The first channel compounds nominal rigidities across industries, conditional on exogenous attention levels, resulting in a flatter Phillips curve. The second channel operates through an optimal adjustment of attention levels in response to the change in the network structure, and has ambiguous effects on the Phillips curve slope. In a quantitative exercise, we find that the endogenous attention channel explains about 30 percent of the attenuated inflation response to monetary shocks, when switching from a horizontal economy to a US-calibrated production economy.

Our empirical analysis draws on the Survey of Firms' Inflation Expectations (SoFIE), a nationally representative quarterly survey of US firms. Figure 1 plots industry-average inflation nowcasts and one-year-ahead forecasts against realized CPI inflation over time. The dispersion across industries is striking in both panels, with some industries tracking the actual rate closely and others deviating substantially.

FIGURE 1. Perceptions about current and future inflation across industries



A. Inflation nowcasts

B. 1-year-ahead inflation forecasts

Note: Panel A: The black line is the CPI inflation rate; the colored dots are industry-average nowcasts of current inflation. Panel B: The horizontal axis is the forecast target date. The black line is realized CPI inflation at that date, and the colored dots are the industry-average one-year-ahead forecasts of it formed one year earlier; so the vertical distance between a dot and the line is that date's forecast error. Industries: Basic Metals, Chemicals and Plastics, Electrical, Financial Intermediation, Food and Drink, Hotels and Restaurants, Mechanical Engineering, Other Manufacturing, Other Services, Post and Telecommunication, Renting and Business Activities, Textiles and Clothing, Timber and Paper, Transport Manufacturing, Transport and Storage Services. Sample from 2Q2018 to 1Q2025. See Section 2 for more details on the survey.

To investigate what drives this cross-industry dispersion in inflation expectations, we combine the SoFIE with data on input-output linkages and industry-level price rigidity. We measure firms' attention to inflation using the absolute value of forecast errors and regress this measure on industry characteristics, including the industry's Domar weight, upstreamness, price flexibility, and the volatility of productivity shocks. We find that firms in industries with larger Domar weights and more flexible prices have significantly more accurate inflation forecasts, while upstreamness and TFP volatility do not have significant effects once we control for these variables.

To rationalize these patterns and study their macroeconomic implications, we embed rational inattention in a dynamic multi-sector price-setting model. Firms produce differentiated goods using labor and intermediate inputs sourced from other industries. The economy is subject to aggregate nominal demand shocks and industry-specific productivity shocks, and firms set prices under incomplete information about these shocks while optimally choosing how to allocate their limited attention. Crucially, the information friction is the source of both heterogeneous expectations and nominal rigidities, so attention and price flexibility are jointly determined.

We first analyze a tractable static case of the model that yields closed-form results. This simplified setting delivers the key analytical insights described below. We then return to the general dynamic model for quantitative analysis.

Firms form beliefs about their industry-specific marginal cost, the relevant variable for their price-setting decision, while considering the costs of acquiring information about the state of the economy. Firms' equilibrium attention is determined by (i) the cost of obtaining information, (ii) the profit losses due to a pricing mistake of a given size, and (iii) the volatility of marginal costs. Importantly, the second and third components are industry-specific, implying that firms in different sectors have varying incentives to acquire information. At the heart of the model is a fixed point between the responsiveness of marginal costs to shocks and attention levels that determine nominal rigidities. Intuitively, low attention in a given industry results in dampened responses of that industry's prices to aggregate and sectoral shocks. This leads to a reduction in the volatility of marginal costs in downstream sectors, thus reducing the benefits of acquiring information and encouraging further inattention within those sectors. As a result, inattention, and hence price rigidity, compounds on the production network due to strategic complementarities in pricing and decisions about information acquisition.

Our framework yields a decomposition of firms' inflation expectations into three components: (i) firms' attention to their industry-specific marginal cost, (ii) the comovement between the marginal cost and aggregate inflation, and (iii) changes in marginal costs. This decomposition arises because firms primarily acquire information about their marginal cost, the relevant variable for price setting, and use this imperfect knowledge to form beliefs about aggregate inflation. A direct corollary connects nowcast accuracy to industry characteristics: nowcast accuracy about aggregate inflation is increasing in both the firm's attention level and the R-squared of a regression of aggregate inflation on industry-level marginal cost changes. Thus, nowcast accuracy depends not only on how attentive firms are, but also on how informative their industry's cost conditions are about aggregate inflation.

Stronger input-output linkages flatten the Phillips curve by compounding nominal rigidities across industries (Basu 1995; Rubbo 2023). Endogenous attention can dampen or reinforce such a flattening of the Phillips curve, depending on what stronger input-output linkages imply for the optimal attention levels in the economy. We show that endogenous attention further flattens the Phillips curve when stronger input-output links sufficiently dampen how much aggregate demand shocks move marginal costs, that is, when the additional compounding of inattention on the production network is strong enough.

To quantify these analytical insights, we consider a dynamic model, calibrated to the US economy using input-output tables, industry-level productivity data, and expectations data from the SoFIE. We first validate the model and show that it replicates the negative relationship between industry-level absolute inflation nowcast errors and Domar weights or price flexibility.

We then study the role of endogenous attention in the propagation of monetary shocks through the production network. Production networks dampen the inflation response while amplifying and prolonging the real output response. Endogenous attention accounts for about 30 percent of the dampening of the inflation response and about 20 percent of the amplification in the output response to monetary shocks.

Related literature. Our model builds on the literature on rational inattention in price-setting problems (Sims 2003; Maćkowiak and Wiederholt 2009, among many others), which has mainly abstracted from input-output linkages. Afrouzi (2024) studies how oligopolistic competition shapes firms' strategic inattention to aggregates, showing that firms with fewer competitors are less attentive. Fang et al. (2024) use EDGAR browsing data to measure firm-to-firm attention allocation in production networks, documenting that firms pay more attention to closer and more volatile network neighbors. Our paper complements both by studying attention to aggregate inflation within a production network, providing a theoretical decomposition of CPI expectations and empirical evidence using inflation forecast errors.

Two other recent contributions share a key mechanism with our paper: firms form beliefs about aggregate conditions based on the realization of their own cost conditions. Minton and Monnery (2025) use US survey data on firm-level cost expectations, documenting over-reaction to idiosyncratic cost movements and under-reaction to aggregate shocks.

Gagliardone and Tielens (2026) use Belgian micro-data linking firm-level prices, costs, and survey expectations to show that firms respond strongly to realized cost shocks but exhibit muted responses to expected future cost growth. In our framework, this learning mechanism arises endogenously from rational inattention, and sectoral differences determine which industries have stronger incentives to attend to their costs.

A substantial body of work examines inflation and monetary policy within production networks, as in, for instance, Pastén, Schoenle, and Weber (2024) and Rubbo (2023). Our model particularly builds upon the approach of La'O and Tahbaz-Salehi (2022), which introduces nominal rigidities to a multi-sector economy through incomplete information in a static setting. However, when firms' attention is endogenous to the production structure, conventional results (such as flatter Phillips curves in economies with stronger intermediate input shares) may no longer hold.

On the empirical side, a growing literature documents departures of firms' inflation expectations from full-information rational expectations (FIRE) predictions and explores their determinants (Coibion, Gorodnichenko, and Kumar 2018; Andrade et al. 2022; Candia, Coibion, and Gorodnichenko 2024; Weber et al. 2025). Notably, Candia, Coibion, and Gorodnichenko (2024), also using the SoFIE, find systematic differences in firms' forecasts across industries. In related work, Hajdini et al. (2025) use a randomized controlled trial of firm pairs in New Zealand to show that macroeconomic expectations propagate along supply chain networks through firm-to-firm communication.

Finally, this paper is connected to a large literature that studies the implications of incomplete information for inflation dynamics and monetary non-neutrality (Lucas 1972; Woodford 2003; Mankiw and Reis 2002; Nimark 2008; Angeletos and Huo 2021, among others). We share with that literature the use of information frictions as the source of nominal rigidities.

Outline. The paper proceeds as follows. Section 2 presents our data and empirical results relating US firms' inflation expectations to industry characteristics. In Section 3, we introduce the dynamic rational inattention model of price-setting in a multi-sector production network. Section 4 characterizes the equilibrium of a tractable static version of the model and derives analytical insights into the determinants and macroeconomic consequences of rational inattention for the production network. Section 5 presents quantitative results from the calibrated dynamic general equilibrium model, evaluates its fit to the cross-industry evidence, and studies the implications of rational inattention for the transmission of monetary shocks. Section 6 concludes.

2. Empirical Results

In this section, we present a number of stylized facts relating the accuracy of firms' inflation expectations to their industry characteristics.

2.1. Data

The Survey of Firms' Inflation Expectations (SoFIE). At the center of our empirical analysis is the firm-level data on inflation expectations sourced from the Survey of Firms' Inflation Expectations (SoFIE). The SoFIE is a nationally representative, quarterly survey of inflation expectations whose results are updated by the Federal Reserve Bank of Cleveland. The survey elicits inflation expectations from CEOs and other top executives and has been conducted since the second quarter of 2018. It relies on the recurring participation of a panel of firms, with between 300 and 700 firms participating each quarter. Importantly, the dataset provides information about each firm, including the sector in which the firm operates (services or manufacturing), its industry, and its size (measured by a categorical classification based on the number of employees).[1]

We make use of two questions that elicit beliefs about aggregate inflation in the survey. The first question, posed every quarter, asks firms to provide their expected consumer price index (CPI) inflation rate over the upcoming 12-month period:

> *What do you think will be the inflation rate (for the Consumer Price Index) over the next 12 months? Please provide an answer in an annual percentage rate.*

The second question we rely on, elicited every third quarter of the year, asks firms about their perceived CPI inflation rate over the last 12 months:

> *What do you think has been the annual inflation rate (for the Consumer Price Index) over the last twelve months? Please provide an answer in annual percentage rate.*

Appendix A.1 provides additional details on the survey methodology and question rotation, following Garciga et al. (2023).[2]

[1] Firms are selected from both the manufacturing and the services sectors. Within the manufacturing sector, companies are classified into food and drink, textiles and clothing, electrical, chemicals and plastics, transport, timber and paper, basic metals, mechanical engineering, and other manufacturing. Within the services sector, the companies are classified into hotels and restaurants, transport and storage, post and telecommunication, financial intermediation, renting and business activities, and other services. The firm size classification is categorical: firms are divided into small (1-19 employees), medium (20-249 employees), and large (250+ employees).

[2] Candia et al. (2026) describe survey questions recently added to SoFIE that we do not make use of in the current paper.

Industry-level data sources. We use three data sources with industry-level information to empirically study how industry characteristics explain differences in firms' inflation expectations. First, we use the 2022 input-output tables constructed by the Bureau of Economic Analysis (BEA) to compute different measures of industry importance. Our second data source is from Pastén, Schoenle, and Weber (2024), who compute the probability of price adjustments at the industry level using the microdata that underlie the Bureau of Labor Statistics (BLS) producer price index (PPI). Third, we rely on the BEA/BLS Integrated Industry-Level Production Account (ILPA) to extract data on industry-specific productivity from 1987 to 2021. Table A1 in the appendix provides additional details on data sources and variable definitions.

2.2. Measurement

Nowcast and forecast accuracy. We calculate the accuracy of firms' inflation nowcasts as the absolute difference between the current annual CPI inflation and each firm's current nowcast. We similarly compute forecast accuracy as the absolute difference between the current annual CPI inflation and each firm's forecast reported a year ago. The following equation summarizes our measure of nowcast or forecast accuracy:

$$|\mathrm{FE}_{fi,t-h}[\pi_t]| \equiv |\pi_t - \mathbb{E}_{fi,t-h}[\pi_t]|, \quad \text{for } h \in \{0, 4\}. \tag{1}$$

Industry importance. Measures of industries' importance in the economy are constructed at the two-digit NAICS industry classification level for consistency with the SoFIE dataset. Specifically, we compute industries' total sales as a fraction of GDP, also known as sales-based Domar weights, and a measure of industry "upstreamness" based on Antràs et al. (2012), which captures the importance of a particular industry as a direct and an indirect supplier to all industries in the economy. Formally, let A be the input-output matrix and $L \equiv (I - A)^{-1}$ be the Leontief inverse matrix. The entry (i, j) of the Leontief inverse matrix L quantifies the direct and indirect dependencies of sector i on sector j. The upstreamness measure is given by

$$u_i \equiv \sum_{k=1}^{n} \ell_{ki}, \tag{2}$$

i.e., it is the i-th column sum of the Leontief inverse (Carvalho and Tahbaz-Salehi 2019).

TFP volatility. We compute the volatility of industry-level productivities after linearly detrending the productivity series from ILPA for each industry.

Marginal costs. We use the BEA input-output data to compute approximate measures of industry-specific marginal costs. Under Cobb-Douglas technologies, Gagliardone et al. (2023) show that an industry's nominal log-marginal cost equals the log of its nominal average variable costs plus a term representing the degree of returns to scale:

$$mc_{it} = \ln(\text{TVC}_{it}/Y_{it}) + \ln(1 + \zeta_i). \tag{3}$$

Using data from the BEA input-output accounts, we compute total variable costs TVC_{it} as the sum of intermediate input and labor costs. We measure real output with the BEA's chain-type real gross output series (2017 dollars).

Industry-specific price rigidity. We aggregate industry-level probabilities of price adjustment measured by Pastén, Schoenle, and Weber (2024) to match the two-digit industry classification in the SoFIE data by calculating a simple average based on the more granular industry classification they have. The resulting measure is the quarterly probability of price adjustment.

2.3. Results

We estimate how industry characteristics correlate with the accuracy of firms' inflation nowcasts and forecasts by running the following regression:

$$|\text{FE}_{fit}[\pi_{t+h}]| = \chi_t + \boldsymbol{\gamma}'\boldsymbol{X}_i + \boldsymbol{\delta}'\boldsymbol{D}_{fit} + \text{error}_{fit}, \quad \text{for } h \in \{0, 4\} \tag{4}$$

where χ_t denotes time fixed effects; $\boldsymbol{X}_i$ is a vector of time-invariant industry characteristics: the industry Domar weight (sales-to-GDP ratio), industry upstreamness, price flexibility (the industry-specific probability of price adjustment), and the volatility of industry-specific productivity shocks; $\boldsymbol{D}_{fit}$ is a vector of firm size dummies (medium, large; small is the reference category). $\boldsymbol{\gamma}$ are the coefficients of interest. Table 1 summarizes the results of this regression.

The coefficient on the Domar weight in regression (4) is negative and statistically significant at the 1 percent level, indicating that firms in industries with larger Domar weights have more accurate inflation nowcasts. The estimated coefficient of −0.831 (column 3) implies that a 1 percentage point increase in an industry's sales-to-GDP ratio is associated with a 0.83 percentage point reduction in absolute nowcast errors. The coefficient on price flexibility is also negative and statistically significant. Since price flexibility is measured

TABLE 1. Forecast errors and industry characteristics

	$\lvert FE_{fit}[\pi_t]\rvert$			$\lvert FE_{fit}[\pi_{t+4}]\rvert$		
	(1)	(2)	(3)	(4)	(5)	(6)
Domar weight	-0.730***	-0.882***	-0.831***	-0.014	-0.260**	-0.218*
	(0.248)	(0.234)	(0.234)	(0.143)	(0.119)	(0.119)
Upstreamness	0.026	0.029	0.025	-0.012	-0.003	-0.009
	(0.043)	(0.040)	(0.040)	(0.023)	(0.019)	(0.019)
TFP volatility	0.001	0.007	0.003	-0.013	-0.006	-0.008
	(0.033)	(0.031)	(0.031)	(0.021)	(0.018)	(0.018)
Price flexibility	-3.487***	-3.816***	-3.635***	-0.505	-1.208**	-1.002**
	(1.020)	(0.963)	(0.961)	(0.577)	(0.473)	(0.475)
Observations	2,178	2,178	2,178	7,884	7,884	7,884
R^2	0.021	0.136	0.139	0.002	0.305	0.309
Time FE	No	Yes	Yes	No	Yes	Yes
Size controls	Yes	No	Yes	Yes	No	Yes

Note: This table reports regressions of absolute nowcast or forecast errors of inflation on industry characteristics. Size controls correspond to firm size dummies (medium, large; reference: small). Robust standard errors in parentheses. ***, **, * indicate statistical significance at 1, 5, and 10 percent, respectively.

as the share of firms within an industry that adjust their price in a given quarter, the coefficient of −3.635 implies that a 10 percentage point increase in an industry's frequency of price adjustment is associated with a 0.36 percentage point reduction in absolute nowcast errors. Across the full range of price flexibility in our sample, the implied difference is approximately 2.6 percentage points.

In contrast, we find that upstreamness and TFP volatility do not have statistically significant effects on nowcast or forecast accuracy once we control for Domar weights and price flexibility. In Appendix A.4, we show that our results are robust to using the cost-based Domar weight as an alternative measure of industry importance.

Next, we regress firms' inflation expectations on changes in our measure of industry-specific marginal costs:

$$\mathbb{E}_{fit}[\pi_{t+h}] = \chi_i + \chi_t + \beta \Delta \mathrm{mc}_{it} + \mathrm{error}_{fit}, \quad \text{for } h \in \{0, 4\}. \tag{5}$$

We guard against potential misspecifications by including industry and time fixed effects. Industry fixed effects absorb level differences in expectations across industries, including

TABLE 2. Inflation expectations and marginal costs

	(1)	(2)	(3)	(4)	(5)	(6)
	Dependent variable:					
	1-year-ahead forecast			Nowcast		
Δmc_{it}	0.107***	0.012***	0.013***	0.152***	0.008	0.010
	(0.0340)	(0.0040)	(0.0034)	(0.0502)	(0.0066)	(0.0062)
Time FE		✓	✓		✓	✓
Industry FE			✓			✓
Observations	7,532	7,532	7,532	1,832	1,832	1,832
R-squared	0.048	0.407	0.411	0.072	0.476	0.480

Note: The table shows the results of regression (5). Δmc_{it} is the year-over-year change in industry log marginal cost, computed as nominal variable cost (intermediate inputs plus employee compensation) divided by real gross output from the BEA Use and Gross Output tables; both Δmc_{it} and expectations are in percentage points. For columns 1-3, the outcome is the firms' 1-year-ahead inflation expectations. For columns 4-6, the outcome is the firms' inflation nowcast. Standard errors are clustered at the industry level. ***, **, * indicate statistical significance at 1, 5, and 10 percent, respectively.

the returns-to-scale term in our marginal-cost measure, while time fixed effects capture aggregate shocks that move both marginal costs and inflation expectations.

The regression results in Table 2 provide evidence that firms' inflation expectations comove with their industry-specific marginal costs. The coefficient on Δmc_{it} is positive and statistically significant for one-year-ahead forecasts, and stable at 0.012–0.013 once fixed effects are included. A 1 percentage point increase in industry marginal-cost growth is associated with a 0.013 percentage point increase in the inflation expectations of firms in that industry, relative to firms in other industries at the same date. The nowcast estimates are of similar magnitude but less precisely estimated in the smaller nowcast subsample. The modest magnitudes follow from what the survey asks: firms forecast economy-wide inflation, and any single industry's cost growth is only weakly informative about it; so even firms that make full use of their cost conditions should attach a limited weight to them. The evidence is thus consistent with firms learning about aggregate inflation through their own cost conditions, in line with Andrade et al. (2022), who document that firms' expectations about aggregate conditions respond persistently to industry-specific conditions.

Taking stock. Before moving on to our model, we summarize our empirical findings and explain what they imply for our modeling choices. Our empirical analysis yields two key results. First, firms' inflation forecasting accuracy depends on the Domar weight and price flexibility of the industry they operate in. Second, firms' inflation expectations are positively associated with the growth rate of the industry-specific marginal cost. These findings imply that firms rely on heterogeneous information sets that depend heavily on industry-specific characteristics when forming inflation expectations. The question we then ask is: How do firms optimally choose the information set that would give rise to our empirical findings in Tables 1 and 2? To answer this question, we focus on a rational inattention model, where firms optimally choose their information sets. As we show in subsequent sections, the model matches the empirical patterns documented above and has important implications for the transmission of monetary shocks to the economy. Two features of the evidence point specifically to a production network. First, the industry characteristic that predicts forecast accuracy, the sales-based Domar weight, is itself a network object. It exceeds the industry's share in final consumption because part of the industry's output is sold as inputs to other industries, and the two coincide in an economy without input-output linkages. Second, linkages make a firm's marginal cost informative about aggregate conditions. The prices firms pay for intermediates are set by other industries responding to the same aggregate shocks, so observing own costs carries economy-wide information.

3. Model

Motivated by the evidence presented in the previous section, we build a multi-sector model of rationally inattentive price-setting firms to rationalize the empirical findings and study their macroeconomic implications.

3.1. Environment

Time is discrete and indexed by $t \in \{0, 1, \ldots\}$.

Intermediate firms. There are n industries in the economy, indexed by $i \in I \equiv \{1, 2, \ldots, n\}$. Each industry comprises a unit mass of monopolistically competitive firms indexed by $f \in [0, 1]$. The output of each industry can be consumed by households or used as an intermediate input for production by firms in all industries, including the same industry.

The monopolistically competitive firms within each industry set prices and use a common constant-returns-to-scale, Cobb-Douglas technology F_i to transform labor and intermediate inputs into differentiated goods:

$$\begin{aligned} Y_{fit} &= Z_{it} F_i(L_{fit}, X_{fit,1}, \dots, X_{fit,n}) \\ &= Z_{it} \zeta_i L_{fit}^{\alpha_i} \prod_{k=1}^{n} X_{fit,k}^{a_{ik}}, \quad \alpha_i + \sum_{k=1}^{n} a_{ik} = 1, \end{aligned} \tag{6}$$

where Y_{fit} is the firm's output, L_{fit} is its labor input, and $X_{fit,k}$ is the amount of sectoral commodity $k \in I$ purchased by the firm. Additionally, Z_{it} is an industry-level productivity shock, and $\zeta_i \equiv \alpha_i^{-\alpha_i} \prod_k a_{ik}^{-a_{ik}}$ is a normalization constant that simplifies the calculations. We assume that industry-level productivities are log-normally distributed according to

$$\log \mathbf{Z}_t \sim \mathcal{N}(0, \Sigma), \tag{7}$$

where $\mathbf{Z}_t \equiv (Z_{it})_{i \in I}$ and Σ is the covariance matrix. The economy's input-output linkages are summarized by the matrix $A \equiv [a_{ij}]_{i,j \in I}$ and industry labor shares by the vector $\boldsymbol{\alpha} \equiv (\alpha_i)_{i \in I}$.

Firms' nominal profits are given by

$$\Pi_{fit} = (1 - \tau_i) P_{fit} Y_{fit} - W_t L_{fit} - \sum_{k=1}^{n} P_{kt} X_{fit,k}, \tag{8}$$

where P_{fit} is the nominal price charged by the firm, P_{kt} is the nominal price of industry k's sectoral output, and τ_i is an industry-specific revenue tax (or subsidy) levied by the government. Differentiated products produced by the unit mass of firms in each industry i are aggregated into a sectoral good using a constant-elasticity-of-substitution (CES) production technology with elasticity of substitution θ_i:

$$Y_{it} = \left(\int_0^1 Y_{fit}^{(\theta_i - 1)/\theta_i} df \right)^{\theta_i/(\theta_i - 1)}. \tag{9}$$

It follows that the demand schedule of firm f in industry i is given by

$$Y_{fit}^d = \left(\frac{P_{fit}}{P_{it}} \right)^{-\theta_i} Y_{it}. \tag{10}$$

Households. The representative household has preferences as in Golosov and Lucas (2007), defined over a consumption aggregate C and total labor supply L:

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\ln C_t - L_t \right) \right], \tag{11}$$

where β is the discount factor. The final consumption basket is given by $C_t = \mathcal{C}(C_{1t}, \ldots, C_{nt})$, where C_{it} is the household's consumption of the good produced by industry i and the consumption aggregator $\mathcal{C}$ is a Cobb-Douglas function expressed as follows:

$$\mathcal{C}(C_{1t}, \ldots, C_{nt}) = \prod_{i=1}^{n} C_{it}^{\gamma_i}, \quad \sum_{i=1}^{n} \gamma_i = 1, \tag{12}$$

where $\boldsymbol{\gamma} \equiv (\gamma_i)_{i=1}^{n}$ is the vector of consumption shares in the household's consumption basket. The representative household's budget constraint is given by

$$P_t C_t + B_t \leq W_t L_t + (1 + i_{t-1}) B_{t-1} + \sum_{i=1}^{n} \int_0^1 \Pi_{fit} df + T_t, \tag{13}$$

where $P_t = \mathcal{P}(P_{1t}, \ldots, P_{nt})$ is the nominal price index implied by the consumption aggregator $\mathcal{C}$, B_t are nominal bond holdings, i_t is the nominal interest rate, and T_t are lump-sum net government transfers to households.

Wage rigidity. We model nominal wage rigidity akin to Blanchard and Galí (2007):

$$W_t = (W_t^*)^{\delta_w} (W_{t-1})^{1-\delta_w}, \tag{14}$$

where $\delta_w \in [0, 1]$ denotes the degree of wage flexibility and W_t^* is the nominal wage in the flexible-price equilibrium.

Government. The government's fiscal instrument is a collection of industry-specific taxes on firms' revenues, with the resulting tax collection then rebated to the representative household as a lump-sum transfer. The government's budget constraint is given by

$$T_t + B_t = (1 + i_{t-1}) B_{t-1} + \sum_{i=1}^{n} \tau_i \int_0^1 P_{fit} Y_{fit} df. \tag{15}$$

We assume that the central bank controls nominal demand $M_t \equiv P_t C_t$ according to:[3]

$$\Delta \log M_t = \rho_m \Delta \log M_{t-1} + \varepsilon_{mt}, \quad \varepsilon_{mt} \sim \mathcal{N}(0, \sigma_m^2). \tag{16}$$

Information structure. Firms set prices under *rational inattention* about nominal demand M_t and sectoral productivities $\boldsymbol{Z}_t$. Given their correct prior determined by shocks' processes (7) and (16), they choose which signals to observe from a rich set of available signals, $\mathbb{S}$, subject to an information processing constraint. Firms acquire an attention capacity $\kappa_{fit} \geq 0$ subject to a linear cost $\omega \times \kappa_{fit}$, where $\omega > 0$ is the information cost parameter.

The timing within each period is as follows. First, firms make their information choices. Shocks and signals are then drawn, and each firm observes the realization of its signals. Given their information sets, firms set prices, demand for each variety is realized, and firms hire labor and intermediate inputs to satisfy demand according to the production function (6). Nominal prices are thus set under incomplete information, while quantity decisions by firms and the representative household are made after observing prices and the realizations of all shocks.

Formally, a strategy for each firm is a sequence of information processing capacities $\{\kappa_{fit}\}_{t=0}^{\infty}$, signal choices $\{S_{fit} \subset \mathbb{S}\}_{t=0}^{\infty}$, and pricing rules $\{P_{fit} : S_{fit}^t \to \mathbb{R}_+\}_{t=0}^{\infty}$, where $S_{fit}^t \equiv S_{fit}^{t-1} \cup S_{fit}$ denotes the firm's cumulative information set at time t, with S_{fit}^{-1} given. Given strategies for all other firms, each firm maximizes the expected discounted value of profits net of information costs:

$$\begin{gathered}
\max_{\{S_{fit}, P_{fit}, \kappa_{fit}\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t M_t^{-1} \left((1-\tau_i) P_{fit} Y_{fit}^d - W_t L_{fit} - \sum_{k=1}^{n} P_{kt} X_{fit,k} - \omega \kappa_{fit} \right) \middle| S_{fit}^{-1} \right] \\
\text{s.t.} \\
Y_{fit}^d = \left(\frac{P_{fit}}{P_{it}} \right)^{-\theta_i} Y_{it} \quad \text{(demand schedule)} \\
\mathcal{I}(S_{fit}; (M_t, (P_{kt})_{k \in I}, \boldsymbol{Z}_t)) \leq \kappa_{fit} \quad \text{(information processing constraint)}
\end{gathered} \tag{17}$$

where $\beta^t M_t^{-1}$ is the household's nominal stochastic discount factor, and $\mathcal{I}(\cdot;\cdot)$ is Shannon's mutual information function, measuring the information that signals S_{fit} provide about the variables relevant to firms' pricing decision: nominal demand M_t, industry prices $\boldsymbol{P}_t$, and sectoral productivities $\boldsymbol{Z}_t$.

[3]This is a standard approach to model monetary policy. See, for example, Golosov and Lucas (2007) and Afrouzi (2024).

Equilibrium. An equilibrium for this economy is an allocation for the household,

$$\Omega_H \equiv \{(C_{it})_{i\in I}, L_t, B_t : \quad t = 0, 1, \ldots\}, \tag{18}$$

a strategy profile for firms given an initial set of signals

$$\Omega_F \equiv \{(\kappa_{fit}, S_{fit} \subseteq \mathbb{S}, P_{fit}, L_{fit}, (X_{fit,k})_{k\in I}, Y_{fit})_{i\in I, f\in[0,1]} : \quad t = 0, 1, \ldots\} \tag{19}$$

and a set of prices

$$\{(P_{fit})_{i\in I, f\in[0,1]}, W_t, i_t : \quad t = 0, 1, \ldots\},$$

such that: (a) given prices and Ω_F, Ω_H solves the household's utility maximization problem; (b) given prices and Ω_H, no firm has an incentive to deviate from Ω_F; (c) $\{M_t \equiv P_t C_t\}_{t=0}^{\infty}$ follows the monetary policy rule (16); (d) the government budget (15) is balanced; (e) labor and goods markets clear.

4. Analytical Insights

To develop analytical results, we analyze a simplified version of the model with $\beta = 0$ and $m_t \sim \mathcal{N}(0, \sigma_m^2)$, so that agents are myopic and shocks are i.i.d. After characterizing firms' optimal pricing and information acquisition decisions in closed form, we study the determinants and macroeconomic consequences of heterogeneous attention across the production network. On the determinants, we show that inattention propagates downstream and derive a decomposition of firms' CPI inflation nowcasts that connects forecast accuracy to industry characteristics. As to consequences, we show that heterogeneous attention alters the aggregate transmission of shocks and the Phillips curve slope.

4.1. Firms' price-setting and information acquisition decisions

Firms face a decision-making problem comprising four choices: (i) determining attention capacity κ_{if}, (ii) selecting signals to observe S_{if}, (iii) setting the price for their differentiated product p_{if}, and (iv) optimizing production inputs $(\boldsymbol{X}_{fit}, L_{fit})$. We will solve the firms' problem backward, focusing on their information acquisition and pricing decisions.

Input choice. When choosing inputs, firms operate under complete information, making decisions once all signals and prices are realized. Their objective at this stage is to minimize costs while meeting the demand for their products Y^d_{fit}, taking wages and input prices as given:

$$\min W_t L_{fit} + \sum_{k=1}^{n} P_{kt} X_{fit,k} \quad s.t.: \quad F_i(L_{fit}, X_{fit,k}) = Y^d_{fit}. \tag{20}$$

Firms' demand for labor and intermediate inputs is given by

$$L_{fit} = Z_{it}^{-1} \alpha_i \left(\prod_{j=1}^{n} P_{jt}^{a_{ij}} \right) W_t^{\alpha_i - 1} Y_{fit} \tag{21}$$

$$X_{fit,k} = Z_{it}^{-1} a_{ik} \left(\prod_{j=1}^{n} P_{jt}^{a_{ij}} \right) \frac{W_t^{\alpha_i}}{P_{kt}} Y_{fit}. \tag{22}$$

Price setting. After solving for the input choice, firm (i, f)'s profit function can be expressed as follows

$$\Pi(P_{fit}, \mathbf{P}_t, Y_{it}, W_t, Z_{it}) = (1 - \tau_i) P_{fit} \left(\frac{P_{fit}}{P_{it}} \right)^{-\theta_i} Y_{it} - Z_{it}^{-1} \left(\prod_{k=1}^{n} P_{kt}^{a_{ik}} \right) W_t^{\alpha_i} \left(\frac{P_{fit}}{P_{it}} \right)^{-\theta_i} Y_{it}, \tag{23}$$

where $\mathbf{P}_t \equiv (P_{it})_{i \in I}$ is the vector of sectoral prices.

To circumvent the complexity of characterizing analytical rational inattention models, we adopt the usual approach of taking a second-order approximation to the firms' information acquisition and price-setting problems around the full-information equilibrium.[4] The approximated firms' profit function is

$$\begin{aligned} \pi(p_{fit}, \mathbf{p}_t, y_{it}, w_t, z_{it}) =& \Pi_1 p_{fit} + \frac{\Pi_{11}}{2} p_{fit}^2 + \Pi_{12}' \mathbf{p} p_{fit} + \Pi_{13} y_i p_{fit} + \Pi_{14} w p_{fit} + \Pi_{15} z_i p_{fit} \\ &+ \text{terms independent of } p_{fit}, \end{aligned}$$

where all the derivatives of the original profit function are evaluated at the full-information equilibrium. A small letter denotes the log-deviation of the corresponding variable from its steady-state value.

After computing the derivatives of the original profit function at the full-information equilibrium, it follows that a firm's optimal price p^*_{fit} is equal to its industry-specific

[4]The full-information equilibrium is efficient since prices are fully flexible and the government's industry-specific taxes undo monopolistic distortions.

marginal cost:

$$p^*_{fit} = \alpha_i w_t + \sum_{k=1}^{n} a_{ik} p_{kt} - z_{it} \equiv mc_{it}. \tag{24}$$

Since all firms within an industry have the same marginal cost (or the same optimal price), hereafter, we will drop the f subscript and denote it by p^*_{it}.

Information acquisition and pricing. Using the second-order approximation, the solution to firms' input choice, and the equilibrium wage condition (14) in log-linear form, $w_t = \delta_w m_t$, the firms' problem (17) reduces to

$$\begin{aligned} \min_{\kappa_{fit}, S_{fit}, p_{fit}(S_{fit})} \; & \frac{1}{2} B_i \mathbb{E}_{fit}\left[(p_{fit}(S_{fit}) - p^*_{it})^2\right] + \omega \kappa_{fit} \\ \text{s.t. :} \quad & p^*_{it} = \alpha_i \delta_w m_t + \sum_{k=1}^{n} a_{ik} p_{kt} - z_{it}, \end{aligned} \tag{25}$$

The coefficient $B_i \equiv |\Pi_{11}| = \theta_i \lambda_i$ is the curvature of the profit function at the full-information equilibrium (Appendix C.1). It equals the product of the elasticity of substitution across varieties, θ_i, and sector i's Domar weight, $\lambda_i \equiv P_i Y_i / (P_t C_t)$. By market clearing, Domar weights satisfy $\boldsymbol{\lambda}' = \boldsymbol{\gamma}'(I - A)^{-1}$.

As in single-agent rational inattention settings (Maćkowiak, Matějka, and Wiederholt 2018; Afrouzi and Yang 2021), it can be shown that in all equilibria, each firm optimally acquires a single signal collinear with its price. We refer to strategies with this property as *recommendation strategies*: $p_{if} = S_{if}, \; S_{if} \in \mathbb{S}$. The intuition is that all alternative strategies are weakly dominated by some feasible recommendation strategy.

Equilibrium prices are given by:

$$p_{fit} = \mu_{fit}\left(\alpha_i \delta_w m_t + \sum_{k=1}^{n} a_{ik} p_{kt} - z_{it}\right) + \nu_{fit}, \quad \nu_{fit} \perp \left(m_t, z_{it}, (S_{i'f't})_{(i',f') \neq (i,f)}\right). \tag{26}$$

In the equation above, $\mu_{fit} \equiv 1 - e^{-2\kappa_{fit}}$ represents the Kalman gain on the signal, where κ_{fit} is the attention capacity. We call μ_{fit} the firm's attention level (to its own marginal cost). The term ν_{fit} represents the noise in prices induced by rational inattention and has the following stochastic properties:

$$\mathbb{E}[\nu_{fit}] = 0, \quad Var[\nu_{fit}] = \mu_{fit}(1 - \mu_{fit}) Var[p^*_{it}]. \tag{27}$$

A proof for the characterization of equilibrium prices above, as well as the feasibility and optimality of recommendation strategies, is provided in Afrouzi (2024). The conditions required by the Afrouzi and Yang (2021) framework are satisfied in our setting: the loss function is quadratic in the pricing error (equation (25)), the state variables are jointly Gaussian, and the full-information optimal price is linear in the shocks. The information acquisition problem (25) is identical among all firms within the same industry and across time, due to the stationarity of the environment. This implies $\mu_{fit} = \mu_i$ for all firms $f \in [0,1]$ in each industry $i \in I$ and for every period $t \in \{0, 1, \ldots\}$.

Integrating equilibrium individual prices (26) over firms $f \in [0,1]$ within each industry, the idiosyncratic noise ν_{fit} averages out, and industry-level prices are collinear with profit-maximizing prices:[5]

$$p_{it} = \mu_i p^*_{it}. \tag{28}$$

Attention capacity. Having characterized the optimal signal structure for a given attention capacity κ_{if}, firms' information acquisition problem reduces to choosing κ_i:

$$\min_{\kappa_i \geq 0} \left\{ \frac{1}{2} e^{-2\kappa_i} \theta_i \lambda_i V^*_i + \omega \kappa_i \right\},$$

where $V^*_i \equiv Var(p^*_{it}) = Var\left(\alpha_i \delta_w m_t + \sum_{k=1}^n a_{ik} p_{kt} - z_{it}\right)$ is the variance of industry i's marginal cost. Since the problem is identical across firms within an industry, we drop the f subscript. The first-order condition yields $e^{-2\kappa_i} \theta_i \lambda_i V^*_i = \omega$, so:

$$\kappa_i = \frac{1}{2} \max \left\{ 0, \ln \left(\frac{\theta_i \lambda_i V^*_i}{\omega} \right) \right\}.$$

We focus on the case where all firms in the economy acquire some positive amount of information, i.e., $\kappa_i > 0$ for all $i \in I$. A sufficient condition is

$$\omega < \theta_i \lambda_i (\alpha_i^2 \delta_w^2 \sigma_m^2 + \sigma_{z,i}^2) \tag{29}$$

[5]Unlike the oligopolistic setting of Afrouzi (2024), where firms strategically attend to competitors' pricing mistakes, here the continuum of firms within each industry implies that idiosyncratic noise averages out of industry prices. Industry prices are therefore pinned down by fundamentals, and firms have no incentive to attend to their competitors' mistakes.

for all $i \in I$, i.e., the cost of information is low enough for all firms operating in the various industries. Then, industry i's equilibrium attention is given by

$$\mu_i = 1 - \frac{\omega}{\theta_i \lambda_i V_i^*}. \tag{30}$$

The equation above captures the trade-off firms face when choosing the optimal attention level. They balance the benefits of more precise information against the cost of attention, ω. The benefits depend on two industry-specific components. The first, $B_i = \theta_i \lambda_i$, reflects the curvature of the profit function with respect to pricing mistakes. Firms in industries with a higher elasticity of substitution (θ_i) or a higher Domar weight (λ_i) face steeper losses from mispricing. The second component, V_i^*, is the variance of marginal costs. Industries facing more volatile marginal costs – whether due to aggregate demand exposure, upstream price transmission, or idiosyncratic productivity shocks – have more to gain from acquiring information. Together, these forces imply that firms in industries where pricing mistakes are costly and marginal costs are volatile will optimally choose higher attention levels.

4.2. Equilibrium characterization

Expressing the equilibrium pricing and optimal attention conditions (26) and (30) at the industry level in matrix form, we obtain the following characterization of the equilibrium:

PROPOSITION 1. *In equilibrium, sectoral prices and attention levels are given by*

$$\boldsymbol{p}_t = \mathrm{M}(I - A\mathrm{M})^{-1}(\boldsymbol{\alpha}\delta_w m_t - \boldsymbol{z}_t) \tag{31}$$

$$\mathrm{M} = I - \omega \, diag(\theta \boldsymbol{\lambda}')^{-1}\left(Var(\mathrm{M}^{-1}\boldsymbol{p}_t) \odot I\right)^{-1}, \tag{32}$$

where $\boldsymbol{p} \equiv (p_i)_{i \in I}$, $\mathrm{M} \equiv diag(\boldsymbol{\mu})$ *and* $\odot$ *denotes the Hadamard product (element-wise matrix multiplication).*

PROOF. See Appendix C.2. □

The equilibrium is characterized by a fixed-point system between the optimal price vector $\boldsymbol{p}_t$ and attention levels $\boldsymbol{\mu}$.

REMARK 1 (Existence and uniqueness). *The fixed-point system* (31)-(32) *admits a unique interior solution under conditions on the information cost* ω *and the network structure* $(A, \boldsymbol{\alpha})$. *Existence follows from the monotonicity of the attention best-response map (see Appendix Lemma A1) and Tarski's fixed point theorem. Uniqueness follows from a contraction mapping argument:*

under sufficient conditions on ω and the network structure, the spectral radius of the Jacobian of the best-response map is strictly less than one. The formal proof, which adapts Kellogg's fixed point theorem to rational inattention models in production networks, is provided in Fang et al. (2024).

We note that the optimal attention levels are interdependent across the production network. Specifically, inattention in one sector propagates downstream, compounding inattention and, as a result, price rigidity along the supply chain (see Appendix Lemma A1). The mechanism operates through the industries' marginal cost volatility V_i^*, which influences optimal attention as in equation (30). When upstream firms are less attentive, their prices respond less to shocks, dampening the downstream firms' input costs and hence their marginal cost volatility. The lower marginal cost volatility reduces incentives for information acquisition, inducing downstream firms to also become less attentive.

Next, we would like to understand what a change in the production network implies for optimal attention. Remark 2 shows that stronger input-output linkages have ambiguous effects on optimal attention levels. However, we show that when the elasticity of any industry's marginal cost variance, driven by aggregate demand only, is sufficiently negative, stronger input-output linkages reduce optimal attention levels. The elasticity of the aggregate-demand component of the marginal cost variance is negative precisely because of inattention compounding through a more interconnected production network.

REMARK 2 (Sufficient condition for a reduction in optimal attention due to stronger input-output linkages). *We consider the following perturbation to the production network: each element of the i-th row of matrix A increases by δ while the labor share of industry i falls by nδ. Appendix Proposition A1 shows that the direct effect of stronger input-output linkages on the optimal attention level of industry j operates through three channels:*

$$\frac{\partial \mu_j}{\partial \delta} = \frac{1-\mu_j}{\delta}\left[\frac{\delta}{\lambda_j}\underbrace{\frac{\partial \lambda_j}{\partial \delta}}_{\text{Domar weight (+)}} + \frac{\delta}{V_j^*}\left(\underbrace{\frac{\partial V_j^*}{\partial \delta}\Big|_{\Sigma=0}}_{\text{Aggregate demand (-)}} + \underbrace{\frac{\partial V_j^*}{\partial \delta}\Big|_{\sigma_m^2=0}}_{\text{Idiosyncratic TFP (+)}}\right)\right]. \tag{33}$$

These three channels are composed of the positive elasticity of j's Domar weight; the negative elasticity of j's aggregate-demand-driven marginal cost variance; and the positive elasticity of j's idiosyncratic-TFP-driven marginal cost variance. A sufficient condition for a positive δ to have a total negative effect on any industry's optimal attention is that the absolute elasticity of the aggregate-demand-driven marginal cost variance with respect to δ exceeds the sum of the

Domar weight elasticity and the elasticity of the idiosyncratic-TFP-driven marginal cost variance with respect to δ for every industry $j \in \{1, 2, \ldots, n\}$ (see the proof of Appendix Proposition A1 for details).

4.3. Determinants of firms' inflation expectations

The equilibrium characterized above determines attention levels and prices as an abstract fixed-point system. To connect these objects to the cross-industry variation in forecast accuracy documented in Section 2.3, we now decompose industry-average CPI inflation nowcasts into three observable components shaped by the production network.

PROPOSITION 2. *Industry-average CPI inflation nowcasts are given by*

$$\bar{\mathbb{E}}_i[\pi_t] = \underbrace{\frac{Cov(p^*_{it}, p^{CPI}_t)}{Var(p^*_{it})}}_{\text{comovement between mc and CPI}} \cdot \underbrace{\mu_i}_{\text{attention to own mc}} \cdot \underbrace{\Delta p^*_{it}}_{\text{realized mc change}}, \tag{34}$$

where $p^{CPI}_t \equiv \boldsymbol{\gamma}' \boldsymbol{p}_t$ is the log CPI and $\pi_t \equiv p^{CPI}_t - p^{CPI}_{t-1}$ is the CPI inflation rate.

PROOF. See Appendix C.3. □

The first term measures how informative industry i's marginal cost is about aggregate prices: industries whose marginal costs comove strongly with the CPI produce more accurate inflation forecasts. The second term, μ_i, is the firm's attention to its own marginal cost, which varies across industries as established above. The third term is the realized change in marginal costs that triggers the forecast. Firm-level forecasts add idiosyncratic noise to the industry-average component in (34). Consistent with our evidence in Table 2, the model implies that firms' inflation expectations respond to the growth rate of their industry-specific marginal cost.

Corollary 1 maps the decomposition in (34) into the unconditional nowcast accuracy and its dispersion across sectors. Since the model is linear and firms observe a signal about their own marginal cost, they form inflation expectations by projecting the CPI price level p^{CPI}_t onto that signal; the forecast-error variance then follows from standard projection.

COROLLARY 1 (Forecast accuracy). *The expected squared forecast error for a firm in industry i is*

$$\mathbb{E}\left[FE^2_{fit}[\pi_t]\right] = \left(1 - \mu_i R^2_i\right) Var(\pi_t), \tag{35}$$

where R_i^2 is the R-squared from a regression of CPI inflation π_t on changes in industry i's prices (Δp_{it}) or marginal costs (Δp_{it}^*).

PROOF. See Appendix C.4. □

Forecast accuracy rises in both the attention level μ_i and the R-squared R_i^2, which measures how much of the variation in aggregate inflation is explained by industry i's marginal cost. We note that the two drivers are equilibrium objects determined jointly and are correlated across industries. The decomposition in Corollary 1 also provides a direct link between the model and the empirical findings documented in Section 2.3. The two primitive drivers of forecast accuracy in (35), μ_i and R_i^2, each have an observable counterpart. μ_i maps to price flexibility, so firms in industries with more flexible prices produce more accurate forecasts. R_i^2 maps to the Domar weight, which can be high for two reasons: a larger consumption share or greater centrality in the input-output network.[6]

4.4. The slope of the Phillips curve

Finally, we derive the implications of heterogeneous attention for the slope of the Phillips curve. Closed-form expressions are available for the case of myopic firms ($\beta = 0$) with persistent monetary shocks ($\rho_m > 0$).

PROPOSITION 3. *If $\beta = 0$ and $\rho_m > 0$, sectoral Phillips curves are given by*

$$\boldsymbol{\pi}_t = \mathcal{B} y_t + (I - \mathcal{V})\bar{\mathbb{E}}_{t-1}[\Delta \boldsymbol{p}_t^*] - \mathcal{V}\left(p_{t-1} + (I-A)^{-1}\boldsymbol{z}_t\right), \tag{36}$$

where $\boldsymbol{\pi}_t \equiv \boldsymbol{p}_t - \boldsymbol{p}_{t-1}$ and $\Delta \boldsymbol{p}_t^* \equiv \boldsymbol{p}_t^* - \boldsymbol{p}_{t-1}^*$ and

$$\begin{aligned} \mathcal{B} &\equiv \frac{\mathrm{M}(I - A\mathrm{M})^{-1}\boldsymbol{\alpha}}{1 - \boldsymbol{\gamma}'\mathrm{M}(I - A\mathrm{M})^{-1}\boldsymbol{\alpha}}, \\ \mathcal{V} &\equiv [\mathrm{M}(I - A\mathrm{M})^{-1} - \mathcal{B}((I-A)^{-1}\boldsymbol{\gamma} - \boldsymbol{\gamma}'\mathrm{M}(I - A\mathrm{M})^{-1})](I - A), \end{aligned} \tag{37}$$

and $y_t \equiv (m_t - \boldsymbol{\gamma}'\boldsymbol{p}_t) - \boldsymbol{\lambda}'\boldsymbol{z}_t$ denotes the output gap.

PROOF. See Appendix C.5. □

The Phillips curve in (36) resembles its counterpart in the full-information production network model of Rubbo (2023), with one key difference: the attention matrix M, which

[6]Appendix B formalizes both channels: it shows that R_i^2 rises with the consumption share γ_i, and characterizes how network centrality shapes the cross-sectional dispersion of R_i^2 in a two-sector economy.

governs nominal rigidities, is endogenous to the economy's input-output structure rather than exogenously imposed. Attention $\mathcal{M}$ interacts with network A in a non-linear way to shape sectoral Phillips curves. Note that sectoral inflation rates depend on sectors' expectations about their own marginal costs, not aggregate inflation. The expectations term appears in (36) due to the anchoring of expectations to prior beliefs induced by the Kalman filter.

Our framework provides a qualification for an important result of the literature on inflation and networks. Several papers that examine price setting in production networks (Basu 1995; Rubbo 2023, among others) find that Phillips curves are flatter in economies with larger intermediate input shares, as network linkages compound nominal frictions present in each industry. In our setting, the effects of changes in the input-output linkages on the Phillips curve slope are ambiguous, since equilibrium attention levels are endogenous to the input-output structure, as discussed in Remark 2. Corollary 2 formalizes this insight.

COROLLARY 2. *The slope of sectoral Phillips curves, $\mathcal{B}$, is weakly increasing in each attention level $(\mu_i)_{i\in I}$. However, the effects of changes in the input-output structure of the economy $(A, \boldsymbol{\alpha})$ on $\mathcal{B}$ are ambiguous.*

PROOF. See Appendix C.6. □

The first part of the corollary is straightforward: higher attention implies that firms' prices respond more to shocks, steepening the Phillips curve. The ambiguity in the second part arises because changes in the input-output structure affect the Phillips curve slope through two mechanisms: (i) compounding of attention on the network, holding industry-specific attention fixed (exogenous attention channel), and (ii) adjustment of attention levels in response to a change in the network structure (endogenous attention channel). The first channel is well-known in the literature (Basu 1995; Rubbo 2023). The second channel is novel, and it occurs because a change in the input-output structure triggers an adjustment of firms' optimal attention levels, that is, of their degree of price flexibility. Pinning down the net effect of stronger input-output linkages on the slope of an industry-specific Phillips curve demands a quantitative exercise. Nevertheless, our discussion in Remark 2 provides an intuition for when one should expect the endogenous attention channel to further flatten the Phillips curve. In particular, stronger input-output links are expected to dampen the Phillips curve slope if the response of industry-specific, aggregate-demand-driven marginal cost variance to the change in the network is sufficiently negative.

5. Quantitative Results

We now move from the tractable static model to the general dynamic framework of Section 3, restoring forward-looking decisions ($\beta > 0$) and persistent monetary shocks ($\rho_m > 0$), to quantify the macroeconomic implications of endogenous inattention in the production network.

5.1. Calibration

Table 3 summarizes our baseline calibration. The model is calibrated to US quarterly data. We calibrate the input-output parameters (intermediate input shares A, labor shares

TABLE 3. Calibrated parameters

Parameter	Description	Value	Source
α	Labor shares		2022 BEA IO table
A	Input expenditure shares		2022 BEA IO table
γ	Consumption shares		2022 BEA IO table
ω	Information cost	0.030	Anchoring to prior (38)
β	Discount factor	$0.96^{1/4}$	annual real rate = 4%
ρ_m	Persistence of NGDP growth	0.265	US NGDP
σ_m	Std of NGDP growth	0.011	US NGDP
δ_w	Wage flexibility	0.30	$Cov(\Delta m, \Delta w)$
Σ	Cov. matrix of sectoral TFP		ILPA/KLEMS
θ	Industry CES	6	20% markup

α, and final consumption shares γ) using the 2022 BEA input-output tables. Industries are aggregated using Domar weights to match the 15-industry classification in the SoFIE data. We estimate the persistence and volatility of nominal demand growth directly from US nominal GDP data, obtaining $\rho_m = 0.265$ and $\sigma_m = 0.011$. The discount factor $\beta = 0.96^{1/4}$ implies an annual real interest rate of 4 percent and is consistent with the value in Afrouzi (2024). The covariance matrix of industry-specific TFP shocks, Σ, is estimated using data from the BEA/BLS Integrated Industry-Level Production Account (ILPA) and KLEMS databases. We set the wage flexibility parameter $\delta_w = 0.30$ to match the covariance between nominal GDP growth and wage growth in US data. This value is consistent with the empirical evidence in Beraja, Hurst, and Ospina (2019). The elasticity of substitution across varieties within each industry is set to $\theta = 6$, implying a steady-state markup of 20 percent. This value of θ is common in the literature (McKay, Nakamura, and Steinsson 2016).

The information cost ω is the key free parameter. Following a similar strategy to that of Afrouzi (2024) and Maćkowiak and Wiederholt (2015), we calibrate ω to match the degree to which firms anchor their inflation forecasts to prior beliefs. Specifically, we estimate the following regression in the SoFIE data:

$$\mathbb{E}_{fit}[\pi_t] = \chi_t + \Gamma \mathbb{E}_{fit-4}[\pi_t] + \text{error}_{fit}. \tag{38}$$

A higher ω reduces signal precision, causing firms to rely more on their priors, which raises the anchoring coefficient Γ. We estimate $\Gamma = 0.242$ and calibrate $\omega = 0.030$ to match this moment. See Appendix B.2 for details.

5.2. Solution method and model validation

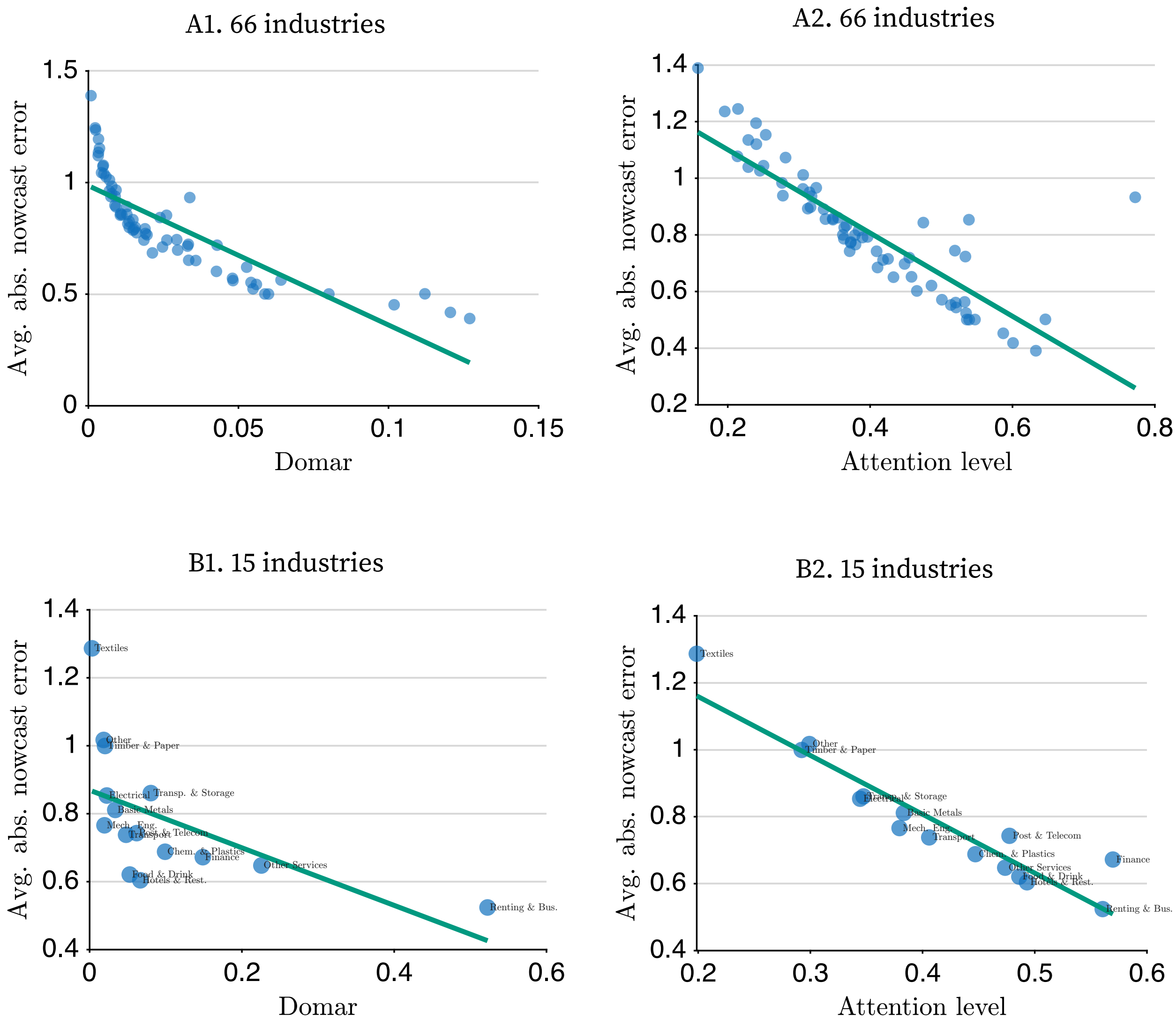
We solve for the stationary equilibrium of the model using an iterative fixed-point algorithm that adapts the method of Afrouzi (2024) to our multi-sector setting with input-output linkages.

The key computational challenge is the non-linear fixed point between sectoral prices and attention levels: prices depend on attention through the degree of nominal rigidity, while optimal attention depends on prices through the volatility of marginal costs. We proceed as follows. Given a guess for the joint Gaussian process of sectoral prices $\boldsymbol{p}_t$, nominal demand m_t, and industry-specific productivities $\boldsymbol{z}_t$, we derive each sector's optimal pricing strategy in a symmetric stationary equilibrium. This determines the stochastic process for sectoral marginal costs $\boldsymbol{p}_t^*$, which we approximate as an integrated moving average in monetary and productivity shocks. We then solve each sector's rational inattention problem using the DRIP method of Afrouzi and Yang (2021), which is efficient enough to permit the many iterations required to calibrate ω. The resulting beliefs and pricing strategies yield an updated joint process for $\boldsymbol{p}_t$, and we iterate until convergence.[7] A detailed description of the solution method is provided in Appendix B.3.

Next, we assess whether the calibrated model can replicate the cross-industry patterns in nowcast accuracy documented in Section 2.3. We simulate time series from the model and regress industry-level average absolute nowcast errors on Domar weights and price flexibility, mirroring the empirical exercise.

[7]As pointed out by Afrouzi (2024), attention capacity is always strictly positive in the dynamic model, unlike in the static case. The unit root in m_t implies that under a zero-capacity strategy, uncertainty about marginal costs grows without bound, so firms always find it optimal to acquire some information.

FIGURE 2. Industry characteristics and forecast accuracy in the model



Note: The figure shows scatter plots of the model-implied relationship between industry-average absolute nowcast errors of inflation and industry characteristics. Each point represents an industry, with the x-axis showing the industry characteristic and the y-axis showing the average absolute nowcast error from simulated data. The top row uses the 66-industry classification from the BEA input-output tables. The bottom row aggregates to the 15-industry classification used in the SoFIE data by computing sales-weighted averages of industry characteristics and forecast errors within each SoFIE industry grouping.

Figure 2 presents scatter plots of model-implied average absolute nowcast errors against industry characteristics. The left panels show that industries with larger Domar weights tend to have smaller forecast errors, consistent with our empirical findings. In the model, industries with higher Domar weights are more central in the production network, so their marginal costs comove more strongly with aggregate prices. As a result, firms in these industries can infer aggregate inflation more accurately from their noisy observation of industry-specific marginal costs. The right panels show that the model captures the positive correlation between price flexibility and forecast accuracy: since the only source of price rigidity in the model is rational inattention, industries with more flexible prices are precisely those with higher attention levels μ_i, which in turn produce more accurate forecasts.

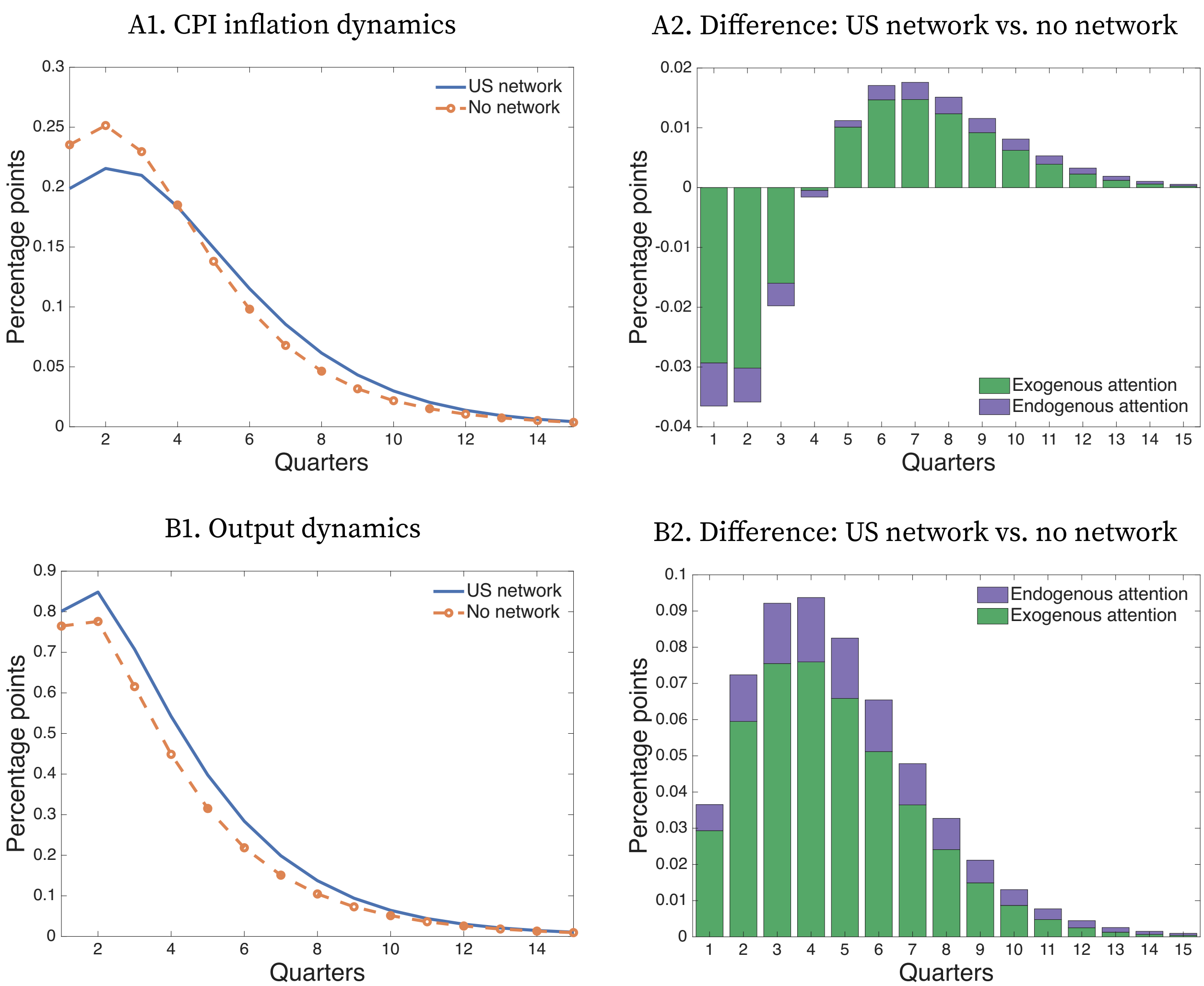
5.3. Propagation of monetary shocks

We now quantify the role of production networks and endogenous attention for the propagation of monetary shocks. In our model, a change in the production network affects the propagation of monetary shocks through two channels. The first, emphasized in the existing literature, operates through real rigidities: input-output linkages create strategic complementarities that compound nominal frictions across sectors. The second, novel to our framework, operates through the endogeneity of sectoral rigidities to the production network: firms' attention levels, and hence the degree of price rigidity in each industry, are shaped by the economy's network structure. The calibrated model allows us to disentangle these two channels.

Figure 3 plots the response of CPI inflation and aggregate output to a 1 percentage point monetary shock in the baseline economy featuring the US production network and in an economy without input-output linkages, that is, a counterfactual horizontal economy. Panel A1 shows that input-output linkages mitigate the impact of an expansionary monetary policy shock on CPI inflation. However, the production network amplifies the response of inflation in the year following the shock.

In our context, the difference between the inflation responses of the two economies confounds the two channels identified above, because firms' attention levels also adjust endogenously to changes in the production network structure. We disentangle these forces. Specifically, we consider an auxiliary economy that features the US production network but holds firms' attention levels fixed at those they would optimally choose in the counterfactual horizontal economy. The gap between this auxiliary economy and the horizontal one isolates the contribution of the production network with exogenous attention levels; the gap between the baseline and the auxiliary economy captures the endogenous attention re-optimization. Panel A2 visualizes the contribution of the first channel in green and of the second channel in purple. We highlight two results. First, the theoretically ambiguous channel of endogenous attention reinforces the exogenous attention channel and further mitigates the response of inflation. The reason is that in this dynamic framework, monetary shocks are persistent and hence central to how firms choose attention levels (see Corollary 2). Second, while the muted response of inflation relative to the horizontal economy is primarily due to the exogenous attention channel, the other channel explains a non-trivial share of it. In particular, the endogenous attention channel accounts for 31 percent, on average, of the muted response of inflation in the first year.

FIGURE 3. Monetary shock transmission: baseline vs. no-network economies



Note: Impulse responses of CPI inflation (panel A1) and output (panel B1) to a 1 percentage point expansionary monetary shock. The baseline economy features the US production network with endogenous attention. The no-network economy has no input-output linkages, but it is subject to endogenous attention. The right panels A2 and B2 decompose the difference in the response between the two economies into the exogenous and endogenous attention channels. To compute the decomposition, we consider the impulse responses of inflation and output of an auxiliary economy that features the US production network but holds attention levels at those of the horizontal economy.

We interpret the inflation impulse responses to be consistent with the production network flattening the slope of the Phillips curve.[8] The flattening of the Phillips curve due to stronger input-output linkages has been argued for in previous papers, such as Rubbo (2023). Our calibrated model provides novel insights and shows that about 30 percent of that flattening is induced by an optimal re-adjustment of nominal rigidities in response to the new production network structure.

On the other hand, panel B1 shows that the production network shifts the response of output to the monetary shock upwards. To understand why that is the case, it is useful to recall that, since monetary policy sets nominal demand, the response of output is described by $y_t = y_{t-1} + \Delta m_t - \pi_t$. The production network initially mitigates the response of inflation, leading to larger real effects of monetary policy. As with inflation, we find that the endogenous channel reinforces the exogenous attention channel of input-output linkages, but it accounts for only 19 percent, on average, of the larger real effects of monetary policy in the first year.[9]

Finally, we note that the contribution of endogenous attention to the response functions of both inflation and output grows with the horizon: as the shock propagates through the network, the information friction compounds, and the attention channel's share of the network effect rises from about a fifth on impact to over half by the end of the 15-quarter horizon. Endogenous attention thus shapes not only the impact of monetary shocks but also the persistence of their transmission through the production network.

6. Conclusion

This paper investigates the causes and macroeconomic consequences of heterogeneity in firms' inflation expectations across industries, documenting new empirical evidence that is rationalized through the lens of a rational inattention model of price-setting firms operating within a production network.

On the empirical side, we combine a comprehensive dataset on US firms' inflation expectations with industry-level data. We find that the accuracy of firms' inflation forecasts is systematically associated with industry attributes: firms in industries with larger Domar weights or more flexible prices have significantly more accurate inflation forecasts. We also find that firms form inflation expectations based on realized industry-specific marginal

[8] We note that we cannot derive an analytical expression of the Phillips curve in this version of the model.

[9] See Appendix Figure A1 for the quarter-by-quarter evolution of the endogenous attention channel's contribution to both CPI inflation and output.

costs, consistent with the view that firms use their own cost conditions as a signal about aggregate inflation.

On the theoretical side, we show that the accuracy of aggregate inflation forecasts varies systematically across industries, depending on both firms' attention to their marginal costs and how informative those costs are about aggregate price movements. Quantitatively, we find that production networks attenuate the inflation response to monetary policy while amplifying and prolonging the output response. The endogenous attention channel accounts for roughly 30 percent of the network's attenuating effect on inflation and about 20 percent of its amplifying effect on output during the first year. As a result, the endogenous attention channel deepens the standard flattening of the Phillips curve from input-output linkages.

References

Afrouzi, Hassan. 2024. "Strategic Inattention, Inflation Dynamics, and the Nonneutrality of Money." *Journal of Political Economy* 132 (10):3378–3420. URL https://doi.org/10.1086/730201.

Afrouzi, Hassan and Choongryul Yang. 2021. "Dynamic Rational Inattention and the Phillips Curve." CESifo Working Paper 8840, CESifo Group Munich. URL https://ssrn.com/abstract=3770462.

Andrade, Philippe, Olivier Coibion, Erwan Gautier, and Yuriy Gorodnichenko. 2022. "No firm is an island? How industry conditions shape firms' expectations." *Journal of Monetary Economics* 125:40–56. URL https://www.sciencedirect.com/science/article/pii/S0304393221000568.

Angeletos, George-Marios and Zhen Huo. 2021. "Myopia and Anchoring." *American Economic Review* 111 (4):1166–1200. URL https://www.aeaweb.org/articles?id=10.1257/aer.20191436.

Antràs, Pol, Davin Chor, Thibault Fally, and Russell Hillberry. 2012. "Measuring the Upstreamness of Production and Trade Flows." *American Economic Review: Papers & Proceedings* 102 (3):412–16. URL https://www.aeaweb.org/articles?id=10.1257/aer.102.3.412.

Basu, Susanto. 1995. "Intermediate Goods and Business Cycles: Implications for Productivity and Welfare." *The American Economic Review* 85 (3):512–531. URL http://www.jstor.org/stable/2118185.

Beraja, Martin, Erik Hurst, and Juan Ospina. 2019. "The Aggregate Implications of Regional Business Cycles." *Econometrica* 87 (6):1789–1833. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA14243.

Blanchard, Olivier and Jordi Galí. 2007. "Real Wage Rigidities and the New Keynesian Model." *Journal of Money, Credit and Banking* 39:35–65. URL http://www.jstor.org/stable/4123055.

Candia, Bernardo, Olivier Coibion, and Yuriy Gorodnichenko. 2024. "The inflation expectations of U.S. firms: Evidence from a new survey." *Journal of Monetary Economics* 145:103569. URL https://www.sciencedirect.com/science/article/pii/S0304393224000229.

Candia, Bernardo, Ina Hajdini, Edward S. Knotek II, Damjan Pfajfar, and Robert W. Rich. 2026. "Expanding the Survey of Firms' Inflation Expectations." Economic Commentary 2026-09, Federal Reserve Bank of Cleveland. URL https://doi.org/10.26509/frbc-ec-202609.

Carvalho, Vasco M. and Alireza Tahbaz-Salehi. 2019. "Production Networks: A Primer." *Annual Review of Economics* 11 (Volume 11, 2019):635–663. URL https://www.annualreviews.org/content/journals/10.1146/annurev-economics-080218-030212.

Coibion, Olivier, Yuriy Gorodnichenko, and Saten Kumar. 2018. "How Do Firms Form Their Expectations? New Survey Evidence." *American Economic Review* 108 (9):2671–2713. URL https://www.aeaweb.org/articles?id=10.1257/aer.20151299.

Fang, Yue, Zhen Huo, Minghao Li, and Jieran Wu. 2024. "Inattentive Network: Evidence and Theory." Working paper, Zhejiang University, Yale University, Peking University.

Federal Reserve Bank of Cleveland. 2023. "Detailed Methodology for the Survey of Firms' Inflation Expectations (SoFIE)." Tech. rep., Federal Reserve Bank of Cleveland.

Gagliardone, Luca, Mark Gertler, Simone Lenzu, and Joris Tielens. 2023. "Anatomy of the Phillips Curve: Micro Evidence and Macro Implications." Working Paper 31382, National Bureau of Economic Research. URL http://www.nber.org/papers/w31382.

Gagliardone, Luca and Joris Tielens. 2026. "Dynamic Pricing under Information Frictions: Evidence from Firm-level Subjective Expectations." URL https://www.lucagagliardone.com/wp-content/uploads/Dynamic_Pricing_under_Information_Frictions-1.pdf. Working Paper.

Garciga, Christian, Edward S. Knotek II, Mathieu Pedemonte, and Taylor Shiroff. 2023. "The Survey of Firms' Inflation Expectations." Economic Commentary 2023-10, Federal Reserve Bank of Cleveland. URL https://doi.org/10.26509/frbc-ec-202310.

Golosov, Mikhail and Robert E. Lucas. 2007. "Menu Costs and Phillips Curves." *Journal of Political Economy* 115 (2):171–199. URL http://www.jstor.org/stable/10.1086/512625.

Hajdini, Ina, Saten Kumar, Samreen Malik, Jordan J. Norris, and Mathieu Pedemonte. 2025. "Supply Chain Networks and the Macroeconomic Expectations of Firms." Working Paper 25-17, Federal Reserve Bank of Cleveland.

La'O, Jennifer and Alireza Tahbaz-Salehi. 2022. "Optimal Monetary Policy in Production Networks." *Econometrica* 90 (3):1295–1336. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA18627.

Lucas, Robert E. 1972. "Expectations and the Neutrality of Money." *Journal of Economic Theory*

4 (2):103–124.

Maćkowiak, Bartosz, Filip Matějka, and Mirko Wiederholt. 2018. "Dynamic rational inattention: Analytical results." *Journal of Economic Theory* 176:650–692. URL https://www.sciencedirect.com/science/article/pii/S002205311830139X.

Maćkowiak, Bartosz and Mirko Wiederholt. 2009. "Optimal Sticky Prices under Rational Inattention." *American Economic Review* 99 (3):769–803. URL https://www.aeaweb.org/articles?id=10.1257/aer.99.3.769.

———. 2015. "Business Cycle Dynamics under Rational Inattention." *The Review of Economic Studies* 82 (4):1502–1532. URL https://ideas.repec.org/a/oup/restud/v82y2015i4p1502-1532..html.

Mankiw, N. Gregory and Ricardo Reis. 2002. "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve." *The Quarterly Journal of Economics* 117 (4):1295–1328. URL https://doi.org/10.1162/003355302320935034.

McKay, Alisdair, Emi Nakamura, and Jon Steinsson. 2016. "The Power of Forward Guidance Revisited." *American Economic Review* 106 (10):3133–3158.

Minton, Robert and Hugo Monnery. 2025. "How Firms Form Beliefs and the Implications for Inflation." Working Paper.

Nimark, Kristoffer. 2008. "Dynamic pricing and imperfect common knowledge." *Journal of Monetary Economics* 55 (2):365–382. URL https://www.sciencedirect.com/science/article/pii/S0304393207001614.

Pastén, Ernesto, Raphael Schoenle, and Michael Weber. 2024. "Sectoral Heterogeneity in Nominal Price Rigidity and the Origin of Aggregate Fluctuations." *American Economic Journal: Macroeconomics* 16 (2):318–52. URL https://www.aeaweb.org/articles?id=10.1257/mac.20210460.

Rubbo, Elisa. 2023. "Networks, Phillips Curves, and Monetary Policy." *Econometrica* 91 (4):1417–1455.

Sims, Christopher A. 2003. "Implications of rational inattention." *Journal of Monetary Economics* 50 (3):665–690.

Weber, Michael, Bernardo Candia, Hassan Afrouzi, Tiziano Ropele, Rodrigo Lluberas, Serafin Frache, Brent Meyer, Saten Kumar, Yuriy Gorodnichenko, Dimitris Georgarakos, Olivier Coibion, Geoff Kenny, and Jorge Ponce. 2025. "Tell Me Something I Don't Already Know: Learning in Low- and High-Inflation Settings." *Econometrica* 93 (1):229–264. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA22764.

Woodford, Michael. 2003. "Imperfect Common Knowledge and the Effects of Monetary Policy." In *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, edited by Philippe Aghion, Roman Frydman, Joseph Stiglitz, and Michael Woodford. Princeton, NJ: Princeton University Press, 25–58.

Appendix A. Additional Empirical Results

This section complements Sections 2.1 and 2.3 by providing more details on the data and additional empirical results.

A.1. Survey of Firms' Inflation Expectations (SoFIE)

The SoFIE encompasses a diverse range of industries, including nine categories within manufacturing and six within the services sector. Furthermore, it categorizes firms based on their employee count, distinguishing between small (1-19 employees), medium (20-249 employees), and large (250+ employees) enterprises. On average, firms engage in the survey for 3.4 waves, with a standard deviation of 3.5. This structured approach ensures a representative sample that captures the perspectives of businesses across various sectors and scales of operation, providing valuable data on inflation expectations within the corporate landscape.

For the second question on inflation beliefs, there is a rotation across four different questions. Each question is asked once per year. In the first quarter, firms are asked about the probability that inflation one year later will exceed 5 percent. In the second quarter, they are asked what inflation rate they think the Federal Reserve is targeting on average. In the third quarter, they are asked what they think the inflation rate has been over the last 12 months. In the fourth quarter of each year, they are asked what they think inflation will be over the next five years on average.

For all questions, respondents are expected to enter a numeric forecast, but the survey instrument accepts ranges (we take the midpoint as the point prediction) or a comment (if possible, we extract a point prediction from the comment). A more detailed description of the survey and its methodology can be found in Federal Reserve Bank of Cleveland (2023).

The relative importance of different sectors and sizes can be adjusted by constructing weights to replicate the distribution of payroll across industries/sizes in the US. Candia, Coibion, and Gorodnichenko (2024) argue that this adjustment does not lead to major differences in their results when doing empirical analysis using the SoFIE data.

A.2. Data sources

Table A1 summarizes the variables used in our empirical analysis, their definitions, and data sources.

TABLE A1. Data sources and variable definitions

Variable	Definition	Source	Period
Inflation expectations	1-year-ahead CPI inflation forecast	SoFIE	2018Q2–
Inflation perceptions	Perceived CPI inflation over past 12 months	SoFIE	2018Q3–
Domar weight	Industry sales as fraction of GDP	BEA I-O Tables	2022
Upstreamness	Column sum of Leontief inverse	BEA I-O Tables	2022
Price flexibility	Probability of price adjustment	Pastén, Schoenle, and Weber (2024)	1998–2014
TFP volatility	Std. dev. of linearly detrended log-TFP	BEA/BLS ILPA	1987–2021
CPI inflation	Consumer price index, 12-month change	BLS	2018–

A.3. Inflation expectations and PPI

Table A2 reports estimates from regressions of firm-level one-year-ahead inflation expectations and inflation nowcasts on industry-level producer price inflation. The regressor ppi_{it} is the year-over-year change in the producer price index for each industry.

TABLE A2. Inflation expectations and PPI

	(1)	(2)	(3)	(4)	(5)	(6)
	Dependent variable:					
	1-year-ahead forecast			Nowcast		
ppi_{it}	0.044***	0.001	0.004**	0.050**	−0.005*	−0.003
	(0.0139)	(0.0018)	(0.0017)	(0.0200)	(0.0029)	(0.0035)
Time FE		✓	✓		✓	✓
Industry FE			✓			✓
Observations	8,908	8,908	8,908	2,178	2,178	2,178
R^2	0.031	0.374	0.377	0.027	0.422	0.428

Note: The table reports regressions of firms' inflation expectations on ppi_{it}, the year-over-year change in the producer price index of the firm's industry, both in percentage points. For columns 1-3, the outcome is the firms' 1-year-ahead inflation expectations. For columns 4-6, the outcome is the firms' inflation nowcast. Standard errors are clustered at the industry level. ***, **, * indicate statistical significance at 1, 5, and 10 percent, respectively.

A.4. Robustness: alternative Domar weight specifications

Table A3 examines the robustness of the main empirical specification in Table 1 to an alternative measure of industry importance: the cost-based Domar weight. The sales-based and cost-based Domar weights are highly correlated in our sample (correlation of 0.96), so we replace the former with the latter rather than including both. The nowcast results are robust to this alternative: the cost-based Domar weight is negatively and significantly associated with absolute nowcast errors once time fixed effects are included, and price flexibility remains strongly associated with more accurate nowcasts. In the forecast-error specifications, the coefficient on the cost-based Domar weight is statistically insignificant, while upstreamness displays a small negative association with forecast errors. TFP volatility is insignificant throughout.

TABLE A3. Forecast errors and industry characteristics (cost-based Domar weight)

	$\lvert FE_{fit}[\pi_t]\rvert$			$\lvert FE_{fit}\pi_{t+4}\rvert$		
	(1)	(2)	(3)	(4)	(5)	(6)
Domar weight (cost)	-0.338	-0.634***	-0.582***	0.228*	-0.036	0.004
	(0.235)	(0.222)	(0.221)	(0.128)	(0.108)	(0.109)
Upstreamness	-0.056	-0.013	-0.017	-0.075***	-0.050***	-0.054***
	(0.035)	(0.033)	(0.033)	(0.020)	(0.017)	(0.017)
TFP volatility	0.024	0.014	0.010	-0.002	-0.004	-0.006
	(0.040)	(0.038)	(0.038)	(0.022)	(0.019)	(0.019)
Price flexibility	-2.008*	-2.987***	-2.758***	0.405	-0.425	-0.189
	(1.031)	(0.963)	(0.965)	(0.561)	(0.459)	(0.463)
Observations	2,178	2,178	2,178	8,581	8,581	8,581
R^2	0.022	0.136	0.139	0.004	0.296	0.300
Time FE	No	Yes	Yes	No	Yes	Yes
Size controls	Yes	No	Yes	Yes	No	Yes

Note: This table reports OLS regressions of absolute forecast errors on industry characteristics. Robust standard errors (HC3) in parentheses. Size controls are dummies for medium and large firms (small is reference). ***, **, * indicate statistical significance at 1, 5, and 10 percent, respectively.

Appendix B. Additional Model Results

B.1. Theoretical analysis

Compounding of inattention on the production network.

LEMMA A1. *Inattention compounds downstream. Concretely, for any sectors $i \neq j$, sector i's best-response attention is weakly increasing in μ_j:*

$$\frac{\partial \mu_i^{BR}(\boldsymbol{\mu})}{\partial \mu_j} \geq 0,$$

where $\mu_i^{BR}(\boldsymbol{\mu})$ denotes sector i's best-response attention level for any sector $i \in I$ given a strategy profile $\boldsymbol{\mu} \in \mathbb{R}^n$.

PROOF. See Appendix C.7. □

B.1.1. Effect of stronger input-output linkages on optimal attention

Next, we would like to understand how a change in the production network structure affects the optimal attention levels. Specifically, we consider the following perturbation: each element of the i-th row of matrix A increases by δ while the labor share of industry i falls by $n\delta$.

PROPOSITION A1. *Consider the above perturbation of the production network of industry i and suppose the conditions for a unique optimal attention vector are satisfied. Then, the direct effect of δ on the optimal attention of industry j is given by:*

$$g_j = \frac{\partial \mu_j}{\partial \delta} = \frac{1-\mu_j}{\delta}\left[\frac{\delta}{\lambda_j}\underbrace{\frac{\partial \lambda_j}{\partial \delta}}_{\textit{Domar weight (+)}} + \frac{\delta}{V_j^*}\left(\underbrace{\frac{\partial V_j^*}{\partial \delta}\Big|_{\Sigma=0}}_{\textit{Aggregate demand (-)}} + \underbrace{\frac{\partial V_j^*}{\partial \delta}\Big|_{\sigma_m^2=0}}_{\textit{Idiosyncratic TFP (+)}}\right)\right]. \tag{A1}$$

Let $\mathbf{g}$ be the vector containing g_j for any $j \in \{1, 2, \ldots, n\}$. The total effect of δ on the vector of optimal attention $\boldsymbol{\mu}$ is

$$\frac{d\boldsymbol{\mu}}{d\delta} = D\mathbf{g}, \tag{A2}$$

where D is a matrix with positive elements, as implied by Lemma A1 and the uniqueness condition in Remark 1.

PROOF. See Appendix C.8. □

An implication of Proposition A1 is that a sufficient condition for stronger input-output links to induce higher nominal rigidities in every industry is that the absolute elasticity of the aggregate-demand-driven marginal cost variance with respect to δ exceeds the sum of the Domar weight elasticity and the elasticity of the idiosyncratic-TFP-driven marginal cost variance with respect to δ for every industry $j \in \{1, 2, \ldots, n\}$.

B.1.2. Determinants of firms' inflation expectations

The next results relate the informativeness R_i^2 to the consumption share and to network centrality, refining the mapping between R_i^2 and the Domar weight discussed in Section 4.3.

LEMMA A2. *Holding attention levels μ fixed, R_i^2 is weakly increasing in the consumption share γ_i, and hence in the Domar weight λ_i through the direct consumption channel. Formally, consider a perturbation that increases γ_i while scaling other consumption shares proportionally so that $\sum_j \gamma_j = 1$ is maintained. Then $\partial R_i^2/\partial \gamma_i \geq 0$. The inequality is strict whenever, in addition, sector i's marginal cost is not perfectly correlated with $\sum_{j \neq i} \gamma_j \mu_j p_{jt}^*$.*

PROOF. See Appendix C.9. □

LEMMA A3. *Consider a two-sector economy with input-output matrix $A = \left(\begin{smallmatrix} 0 & b \\ a & 0 \end{smallmatrix}\right)$, $a, b \geq 0$ and $\bar{\mu}^2 ab < 1$; equal consumption shares $\gamma_1 = \gamma_2 = 1/2$; common attention $\mu_1 = \mu_2 = \bar{\mu} \in (0, 1)$; and equal TFP variances $\sigma_{z,1}^2 = \sigma_{z,2}^2 = \sigma_z^2$. If $a > b$, then*

$$R_1^2 > R_2^2 \iff \frac{\sigma_m^2}{\sigma_z^2} > T(a, b) \equiv \frac{\bar{\mu}^2(a+b)}{(1-\bar{\mu})\left[2 - a - b + \bar{\mu}(a + b - 2ab)\right]\delta_w^2}. \tag{A3}$$

PROOF. See Appendix C.10. □

Under $a > b$, sector 1 is the more central sector: $\lambda_1 - \lambda_2 = (a-b)/[2(1-ab)] > 0$. With symmetric γ and μ, the CPI is (up to a scalar) $p_1^* + p_2^*$, so the sector whose marginal cost varies more contributes a larger share of CPI variation and earns the higher R^2: $\text{sign}(R_1^2 - R_2^2) = \text{sign}(V_1^* - V_2^*)$. Two forces push $V_1^* - V_2^*$ in opposite directions. The monetary channel, with sign $(a-b)(1-\bar{\mu}) \geq 0$, favors sector 1: sector 1's labor share loads directly on aggregate demand, while sector 2's exposure transits through its upstream supplier and is attenuated by the attention filter $\bar{\mu} < 1$. The productivity channel, with sign $-(a^2 - b^2)\bar{\mu}^2 \leq 0$, favors sector 2: upstream TFP shocks reach sector 2's marginal cost through the network,

TABLE A4. Anchoring coefficient: nowcast on lagged 1-year expectation

	$E_t[\pi_t]$
$E_{t-4}[\pi_{t+4}]$	0.242^{***}
	(0.051)
Observations	750
R^2	0.062

Firm-clustered standard errors in parentheses
Time fixed effects included
*** $p < 0.01$

so sector 2 aggregates both sectors' own-productivity variation while sector 1 carries only its own. The threshold $T(a, b)$ separates the two regimes: above it, the monetary channel dominates and $R_1^2 > R_2^2$; below it, the productivity channel dominates and $R_2^2 > R_1^2$.

COROLLARY A1. *At any (a, b) with $a > b$, holding σ_m^2/σ_z^2 and $\bar{\mu}$ fixed:*

(i) $\partial T/\partial b > 0$: as b falls with a fixed, $R_1^2 > R_2^2$ becomes more likely.

(ii) $\partial T/\partial a > 0$: as a rises with b fixed, $R_1^2 > R_2^2$ becomes less likely.

(iii) Along $da = -db > 0$ (widening the centrality gap $a - b$ at fixed network intensity $a + b$), T strictly falls, and $R_1^2 > R_2^2$ becomes more likely.

PROOF. See Appendix C.10. □

Points (i) and (iii) are driven by the monetary channel: sector 1's direct loading on aggregate demand is attenuated in sector 2 by the attention filter, and widening the centrality gap amplifies this asymmetry. Point (ii) is driven by the productivity channel: as a rises, sector 2 absorbs more of sector 1's TFP variation, aggregating both sectors' own-productivity shocks into its marginal cost.

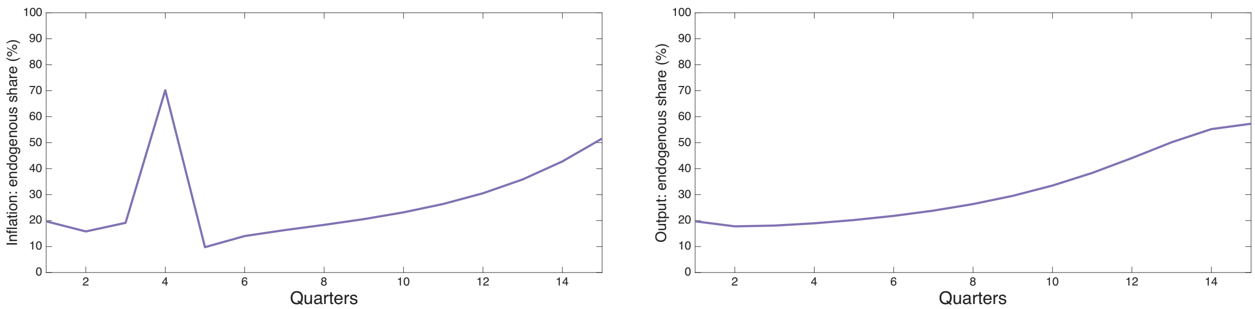
B.2. Quantitative analysis

Calibration of the information cost parameter ω. Table A4 reports the regression results used to calibrate the information cost parameter ω. Following Afrouzi (2024), we estimate the degree to which firms anchor their inflation forecasts to their prior beliefs. The anchoring coefficient measures how much weight firms place on their prior beliefs when forming current expectations. A higher anchoring coefficient indicates greater reliance on prior beliefs, which corresponds to a higher information cost in the model.

Decomposition of the network effect. Figure A1 plots the contribution of the endogenous attention channel to the difference between the US network and no-network responses for CPI inflation and aggregate output.

FIGURE A1. Evolution of the relative importance of the endogenous attention channel

A1. CPI inflation A2. Output



Note: Evolution of the contribution of the endogenous attention channel to the difference between the US network and no-network responses for CPI inflation (panel A1) and output (panel A2).

B.3. Solution method

This section describes the numerical algorithm used to solve for the equilibrium of the dynamic model. The solution is a fixed point between the stochastic process for sectoral prices and firms' optimal attention allocation.

Solution representation. The equilibrium solution takes the form of impulse response functions of sectoral prices to monetary and productivity shocks:

$$\boldsymbol{p}_t = \Phi_m(L)\varepsilon_{mt} + \Phi_z(L)\boldsymbol{z}_t, \tag{A4}$$

where $\Phi_m(L)$ and $\Phi_z(L)$ are lag polynomials capturing the dynamic response of prices to shocks.

Since the monetary policy shock m_t follows a unit root process, we work with the cumulated monetary shock $\tilde{\varepsilon}_{mt} \equiv \sum_{s=0}^{\infty} \varepsilon_{m,t-s} = (1-L)^{-1}\varepsilon_{mt}$. Truncating to a finite lag length $\bar{L} = 40$, the price vector can be approximated as:

$$\boldsymbol{p}_t \approx \Psi x_t, \tag{A5}$$

where Ψ is an $n \times (n+1)\bar{L}$ matrix and x_t is the state vector:

$$x_t \equiv \left(\tilde{\varepsilon}_{mt}, \tilde{\varepsilon}_{m,t-1}, \ldots, \tilde{\varepsilon}_{m,t-\bar{L}+1}, z_{1,t}, \ldots, z_{1,t-\bar{L}+1}, \ldots, z_{n,t}, \ldots, z_{n,t-\bar{L}+1}\right)'.$$

Algorithm. The algorithm iterates between solving firms' rational inattention problems and updating the equilibrium price process:

a. **Initialize**: Start with an initial guess $\Psi^{(0)}$ for the price coefficients.
b. **Solve rational inattention problems**: Given guess $\Psi^{(k)}$, solve the dynamic rational inattention problem (DRIP) for each sector $i \in \{1, \ldots, n\}$.
c. **Update price coefficients**: Using the solutions from step 2, compute the new price impulse responses and update $\Psi^{(k+1)}$.
d. **Check convergence**: If $\|\Psi^{(k+1)} - \Psi^{(k)}\| < \epsilon$, stop. Otherwise, return to step 2. We set $\epsilon = 10^{-4}$.

Model's state-space representation. Given a guess $\Psi^{(k)}$, the state-space representation for the problem faced by firms in industry i is:

$$x_t = Bx_{t-1} + Qu_t, \tag{A6}$$

$$p^*_{it} = H_i^{(k)} x_t, \tag{A7}$$

where $u_t = (\varepsilon_{mt}, \varepsilon_{z_1,t}, \ldots, \varepsilon_{z_n,t})'$ is the vector of innovations with $u_t \sim \mathcal{N}(0, I_{n+1})$.

The transition matrix $B \in \mathbb{R}^{(n+1)\bar{L}\times(n+1)\bar{L}}$ has block structure:

$$B = \begin{bmatrix} B^m & 0 \\ 0 & B^z \end{bmatrix},$$

where $B^m \in \mathbb{R}^{\bar{L}\times\bar{L}}$ captures the dynamics of the cumulated monetary shock and $B^z \in \mathbb{R}^{n\bar{L}\times n\bar{L}}$ captures the productivity shock dynamics.

The observation equation (A7) links the state to firm i's optimal price:

$$H_i^{(k)} = \alpha_i \Psi_m + \sum_{j=1}^{n} a_{ij} \Psi_j^{(k)} - \Psi_{z_i}, \tag{A8}$$

where Ψ_m denotes the rows of Ψ corresponding to monetary shocks, $\Psi_j^{(k)}$ corresponds to sector j's price, and Ψ_{z_i} corresponds to sector i's productivity shock.

Solution to the dynamic rational inattention problem. For each sector i, the dynamic rational inattention problem is:

$$\min_{\{a_{it}\}_{t\geq 0}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left((a_{it} - H_i^{(k)} x_t)^2 + \omega \mathcal{I}(a_{it}; x_t \mid a_{i,t-1})\right)\right], \tag{A9}$$

subject to the state transition (A6) and the marginal cost process (A8).

The solution to this problem yields three objects: (i) K_i, the Kalman gain vector, which determines how firms in industry i update beliefs about x_t after receiving a signal; (ii) Y_i, the loading of optimal signals on the state x_t; and (iii) $\sigma^2_{i,e}$, the variance of the rational inattention noise. Specifically, optimal signals take the form $S_{fit} = Y_i' x_t + e_{fit}, \quad e_{fit} \sim \mathcal{N}(0, \sigma^2_{i,e})$, and the update of beliefs given a signal realization is determined by K_i.

Updating the price coefficients. With the solution to the DRIP, we construct the impulse response functions of prices. If p_{it} responds to shock $\varepsilon_{x,t-\tau}$ with coefficient $w_{i,x,\tau}$, the updated guess for Ψ is:

$$\psi^{(k+1)}_{i,m,\tau} = w_{i,m,\tau}, \tag{A10}$$

$$\psi^{(k+1)}_{i,z_j,\tau} = w_{i,z_j,\tau}, \tag{A11}$$

for all sectors i, shock types, and lags $\tau = 0, \ldots, \bar{L} - 1$. The algorithm then returns to step 2 and iterates until the price coefficients converge.

Appendix C. Proofs

C.1. Derivation of profit function curvature

PROOF. The firm's nominal profit and its CES demand are

$$\Pi_{fit} = (1 - \tau_i) P_{fit} Y_{fit} - MC_{it} Y_{fit}, \qquad Y_{fit} = (P_{fit}/P_{it})^{-\theta_i} Y_{it}.$$

Working in log deviations $p \equiv \log P_{fit}$, and letting $h(p)$ denote demand as a function of p, we can write $\Pi(p) = [(1 - \tau_i)e^p - MC_{it}]h(p)$. Differentiating:

$$\frac{d\Pi}{dp} = h(p)\left[(1 - \tau_i)e^p(1 - \theta_i) + \theta_i MC_{it}\right].$$

At the efficient equilibrium, the industry-specific subsidy satisfies $(1-\tau_i) = \theta_i/(\theta_i - 1)$, which eliminates the monopolistic markup so that $P_{it} = MC_{it}$ and all firms within an industry set the same price. The first-order condition $d\Pi/dp = 0$ is verified at $P_{fit} = P_{it}$. Differentiating again and evaluating at the symmetric equilibrium (where $d\Pi/dp = 0$):

$$\left.\frac{d^2\Pi}{dp^2}\right|_{p=p_{it}} = h(p_{it}) \cdot (1-\tau_i)P_{it}(1-\theta_i) = Y_{it} \cdot \frac{\theta_i}{\theta_i - 1} \cdot P_{it} \cdot (1-\theta_i) = -\theta_i P_{it} Y_{it}.$$

Normalizing by aggregate nominal expenditure $M_t = P_t C_t$, the curvature of the profit function is $B_i \equiv |d^2\Pi/dp^2|/M_t = \theta_i \cdot P_{it}Y_{it}/(P_t C_t) = \theta_i \lambda_i$. □

C.2. Proof of Proposition 1

PROOF. Writing the condition determining marginal costs (24) in vector form yields:

$$\boldsymbol{p}_t^* = \boldsymbol{\alpha}\delta_w m_t + A\boldsymbol{p}_t - \boldsymbol{z}_t. \tag{A12}$$

The equilibrium condition for industry prices in vector form is $\boldsymbol{p}_t = \mathrm{M}\boldsymbol{p}_t^*$ by (26). Substituting that in the marginal cost condition above, it follows that

$$\boldsymbol{p}_t = \mathrm{M}(I - A\mathrm{M})^{-1}(\boldsymbol{\alpha}\delta_w m_t - \boldsymbol{z}_t). \tag{A13}$$

Writing the optimal attention condition (30) in matrix form yields

$$\mathrm{M} = I - \omega \mathrm{diag}(\theta\boldsymbol{\lambda}')^{-1}(\mathrm{Var}(\mathrm{M}^{-1}\boldsymbol{p}_t) \odot I)^{-1}. \tag{A14}$$

□

C.3. Proof of Proposition 2

PROOF. Since the equilibrium is a recommendation strategy, each firm's price equals its signal at time t. Projecting the CPI onto each firm's price, it follows that

$$p_t^{CPI} = \zeta_{if} p_{fit} + u_{fit}.$$

where u_{fit} is a residual term orthogonal to p_{fit} and $\zeta_{if} \equiv Cov(p_{fit}, p_t^{CPI})/Var(p_{fit})$. Using the equilibrium price condition (26), it follows that

$$\begin{aligned} \zeta_{if} &= \frac{Cov(\mu_i p_{it}^* + \nu_{fit}, p_t^{CPI})}{Var(\mu_i p_{it}^* + \nu_{fit})} \\ &= \frac{Cov(p_{it}^*, p_t^{CPI})}{Var(p_{it}^*)} \equiv \zeta_i^*, \end{aligned}$$

where we used the fact that $\mathbb{E}[\nu_{fit}] = 0$, $Cov(\nu_{fit}, p_{it}^*) = 0$, and $Var(\nu_{fit}) = \mu_i(1-\mu_i)Var(p_i^*)$. The last equality follows from the signal structure: since $p_{fit} = \mu_i p_{it}^* + \nu_{fit}$ with ν_{fit} orthogonal to p_{it}^*, we have $Var(p_{fit}) = \mu_i^2 Var(p_{it}^*) + Var(\nu_{fit})$. Under the optimal signal structure, $Var(p_{fit}) = \mu_i Var(p_{it}^*)$, which implies $Var(\nu_{fit}) = \mu_i(1-\mu_i)Var(p_{it}^*)$. Therefore, the firm-level CPI forecast can be expressed as:

$$\mathbb{E}_{fit}[p_t^{CPI}] = \zeta_{if} p_{fit} = \frac{Cov(p_{it}^*, p_t^{CPI})}{Var(p_{it}^*)} \mu_i p_{it}^* + \frac{Cov(p_{it}^*, p_t^{CPI})}{Var(p_{it}^*)} \nu_{fit},$$

where ν_{fit} is a term orthogonal to p_{it}^*. To obtain the industry-average CPI forecast, we take the expectation over all firms within industry i. Since ν_{fit} is orthogonal to p_{it}^* and has zero mean, it drops out when taking expectations:

$$\bar{\mathbb{E}}_{it}[p_t^{CPI}] = \frac{Cov(p_{it}^*, p_t^{CPI})}{Var(p_{it}^*)} \mu_i p_{it}^*.$$

Since the analytical results are derived under i.i.d. shocks, p_{t-1}^{CPI} is independent of current-period signals, so $\bar{\mathbb{E}}_{it}[\pi_t] = \bar{\mathbb{E}}_{it}[p_t^{CPI}]$. Similarly, $\Delta p_{it}^* = p_{it}^*$ since p_{it-1}^* depends only on previous-period shocks with zero unconditional mean. The result follows. □

C.4. Proof of Corollary 1

PROOF. The CPI forecast error for firm f in industry i is $FE_{fit}[\pi_t] = \pi_t - \mathbb{E}_{fit}[\pi_t]$. Since equilibrium prices satisfy $p_{fit} = \mu_i p_{it}^* + \nu_{fit}$ with $\nu_{fit} \perp (p_{it}^*, \pi_t)$, the firm's CPI forecast is a linear projection of π_t onto p_{fit}:

$$\mathbb{E}_{fit}[\pi_t] = \frac{Cov(p_{fit}, \pi_t)}{Var(p_{fit})} p_{fit}.$$

Under the optimal signal structure, $Var(p_{fit}) = \mu_i Var(p^*_{it})$. Using orthogonality of ν_{fit}, $Cov(p_{fit}, \pi_t) = \mu_i Cov(p^*_{it}, \pi_t)$. The R-squared of this projection is therefore

$$\frac{[Cov(p_{fit}, \pi_t)]^2}{Var(p_{fit})Var(\pi_t)} = \frac{\mu_i^2[Cov(p^*_{it}, \pi_t)]^2}{\mu_i Var(p^*_{it})Var(\pi_t)} = \mu_i R_i^2,$$

where $R_i^2 \equiv [Cov(p^*_{it}, \pi_t)]^2/[Var(p^*_{it})Var(\pi_t)]$. The expected squared forecast error is then $\mathbb{E}[FE^2_{fit}] = (1 - \mu_i R_i^2)Var(\pi_t)$. □

C.5. Proof of Proposition 3

PROOF. Under recommendation strategies, firms equate their price with the chosen signal, $p_{fit} = S_{fit}$. With $\beta = 0$, the firm's problem reduces to a one-period quadratic loss around p^*_{it}, and the optimal signal is a Gaussian linear observation of p^*_{it} with Kalman gain μ_i. The posterior mean is a convex combination of the industry-common prior and the realized signal, so

$$p_{fit} = \mu_i p^*_{it} + (1 - \mu_i)\mathbb{E}_{fit-1}[p^*_{it}] + \nu_{fit}, \tag{A15}$$

where $\mathbb{E}_{fit-1}$ denotes the firm's prior belief formed with information through $t-1$, and ν_{fit} is the rational-inattention noise of (26).

Integrating (A15) over firms $f \in [0, 1]$ within each industry, the noise ν_{fit} averages out. Stacking industries,

$$\boldsymbol{p}_t = \mathrm{M}\boldsymbol{p}^*_t + (I - \mathrm{M})\bar{\mathbb{E}}_{t-1}[\boldsymbol{p}^*_t]. \tag{A16}$$

This generalizes (28): the second term captures the anchoring of industry prices to prior beliefs induced by the Kalman filter.

Recommendation strategies imply a useful identity: after integrating out firm-specific noise, the industry's period-$(t-1)$ posterior belief about its own period-$(t-1)$ marginal cost equals the realized industry price, $\bar{\mathbb{E}}_{t-1}[\boldsymbol{p}^*_{t-1}] = \boldsymbol{p}_{t-1}$. Subtracting (A16) at $t-1$ from (A16),

$$\boldsymbol{\pi}_t = \mathrm{M}\Delta\boldsymbol{p}^*_t + (I - \mathrm{M})\big(\bar{\mathbb{E}}_{t-1}[\boldsymbol{p}^*_t] - \bar{\mathbb{E}}_{t-2}[\boldsymbol{p}^*_{t-1}]\big).$$

Decomposing $\bar{\mathbb{E}}_{t-1}[\boldsymbol{p}^*_t] - \bar{\mathbb{E}}_{t-2}[\boldsymbol{p}^*_{t-1}] = \bar{\mathbb{E}}_{t-1}[\Delta\boldsymbol{p}^*_t] + (\boldsymbol{p}_{t-1} - \bar{\mathbb{E}}_{t-2}[\boldsymbol{p}^*_{t-1}])$ and applying (A16) at $t-1$, the second bracketed term equals $\mathrm{M}(I - \mathrm{M})^{-1}(\boldsymbol{p}^*_{t-1} - \boldsymbol{p}_{t-1})$, so $(I - \mathrm{M})$ times it equals

$\mathcal{M}(\boldsymbol{p}^*_{t-1} - \boldsymbol{p}_{t-1})$. Collecting terms gives the law of motion

$$\boldsymbol{\pi}_t = \mathcal{M}\boldsymbol{p}^*_t + (I - \mathcal{M})\bar{\mathbb{E}}_{t-1}[\Delta \boldsymbol{p}^*_t] - \mathcal{M}\boldsymbol{p}_{t-1}. \tag{A17}$$

Following Rubbo (2023), we treat labor as an $(n+1)$-th sector that does not purchase intermediate inputs. Its target price is the nominal wage anchor $w^*_t = m_t$, and it is subject to attention δ_w. Define the augmented objects

$$\hat{\boldsymbol{p}}_t \equiv \begin{pmatrix} \boldsymbol{p}_t \\ w_t \end{pmatrix}, \quad \hat{\boldsymbol{p}}^*_t \equiv \begin{pmatrix} \boldsymbol{p}^*_t \\ m_t \end{pmatrix}, \quad \hat{\mathcal{M}} \equiv \begin{pmatrix} \mathcal{M} & 0 \\ 0 & \delta_w \end{pmatrix}, \quad \hat{A} \equiv \begin{pmatrix} A & \boldsymbol{\alpha} \\ \boldsymbol{0}' & 0 \end{pmatrix}, \quad \hat{\boldsymbol{\alpha}} \equiv \begin{pmatrix} \boldsymbol{0} \\ 1 \end{pmatrix}, \quad \hat{\boldsymbol{z}}_t \equiv \begin{pmatrix} \boldsymbol{z}_t \\ 0 \end{pmatrix},$$

so that $\hat{\boldsymbol{p}}^*_t = \hat{A}\hat{\boldsymbol{p}}_t + \hat{\boldsymbol{\alpha}}m_t - \hat{\boldsymbol{z}}_t$: goods sectors' targets reduce to (24), while the labor sector's target is $w^*_t = m_t$. Steps 2–3 extend unchanged, giving the augmented law of motion $\hat{\boldsymbol{\pi}}_t = \hat{\mathcal{M}}\hat{\boldsymbol{p}}^*_t + (I - \hat{\mathcal{M}})\bar{\mathbb{E}}_{t-1}[\Delta\hat{\boldsymbol{p}}^*_t] - \hat{\mathcal{M}}\hat{\boldsymbol{p}}_{t-1}$. Substituting $\hat{\boldsymbol{p}}^*_t = \hat{A}\hat{\boldsymbol{p}}_t + \hat{\boldsymbol{\alpha}}m_t - \hat{\boldsymbol{z}}_t$ and using $\hat{\mathcal{M}}(\hat{A} - I) = (I - \hat{\mathcal{M}}) - (I - \hat{\mathcal{M}}\hat{A})$ yields the augmented Phillips curve

$$\hat{\boldsymbol{\pi}}_t = (I - \hat{\mathcal{V}})\bar{\mathbb{E}}_{t-1}[\Delta\hat{\boldsymbol{p}}^*_t] - \hat{\mathcal{V}}\hat{\boldsymbol{p}}_{t-1} + \hat{\mathcal{Z}}\hat{\boldsymbol{\alpha}}m_t - \hat{\mathcal{Z}}\hat{\boldsymbol{z}}_t, \tag{A18}$$

with $\hat{\mathcal{V}} \equiv I - (I - \hat{\mathcal{M}}\hat{A})^{-1}(I - \hat{\mathcal{M}})$ and $\hat{\mathcal{Z}} \equiv (I - \hat{\mathcal{M}}\hat{A})^{-1}\hat{\mathcal{M}}$.

Projecting (A18) onto the n goods rows and substituting $m_t = y_t + \boldsymbol{\gamma}'\boldsymbol{p}_t + \boldsymbol{\lambda}'\boldsymbol{z}_t$ introduces a $\boldsymbol{\gamma}'\boldsymbol{p}_t = \boldsymbol{\gamma}'\boldsymbol{p}_{t-1} + \boldsymbol{\gamma}'\boldsymbol{\pi}_t$ term. Moving $\boldsymbol{\gamma}'\boldsymbol{\pi}_t$ to the left-hand side produces the rank-one update $I - u\boldsymbol{\gamma}'$ with $u = \mathcal{M}(I - A\mathcal{M})^{-1}\boldsymbol{\alpha}$; inverting via Sherman–Morrison yields $\mathcal{B} = u/(1 - \boldsymbol{\gamma}'u)$ and $\mathcal{V}$ as in (37). Combining the $\boldsymbol{p}_{t-1}$, $\boldsymbol{z}_t$, and $\boldsymbol{\lambda}'\boldsymbol{z}_t$ contributions gives the residual $-\mathcal{V}(\boldsymbol{p}_{t-1} + (I - A)^{-1}\boldsymbol{z}_t)$ of (36). □

C.6. Proof of Corollary 2

PROOF. By the Neumann series expansion, $(I - A\mathcal{M})^{-1} = \sum_{k=0}^{\infty}(A\mathcal{M})^k$, which is elementwise weakly increasing in $\boldsymbol{\mu}$ since each term $(A\mathcal{M})^k$ has non-negative entries that are weakly increasing in $\boldsymbol{\mu}$. It follows that the numerator $\mathcal{M}(I - A\mathcal{M})^{-1}\boldsymbol{\alpha}$ is elementwise weakly increasing in $\boldsymbol{\mu}$. For the denominator, note that $\boldsymbol{\gamma}'\mathcal{M}(I - A\mathcal{M})^{-1}\boldsymbol{\alpha}$ equals the aggregate price pass-through of a monetary shock, which is strictly less than one under incomplete information ($\mathcal{M} \neq I$). Hence the denominator $1 - \boldsymbol{\gamma}'\mathcal{M}(I - A\mathcal{M})^{-1}\boldsymbol{\alpha}$ is strictly positive and weakly decreasing in $\boldsymbol{\mu}$. Since $\mathcal{B}$ in (37) is the ratio of an increasing numerator to a positive decreasing denominator, $\mathcal{B}$ is weakly increasing in each μ_i. □

C.7. Proof of Lemma A1

PROOF. The covariance matrix of industry-specific marginal costs is given by:

$$Var(\boldsymbol{p}_t^*) = \tilde{L}\Sigma_0\tilde{L}', \tag{A19}$$

where $\tilde{L} \equiv (I - AM)^{-1}$ and $\Sigma_0 \equiv \boldsymbol{\alpha}\boldsymbol{\alpha}'\delta_w^2\sigma_m^2 + \Sigma$. We proceed in three steps.

Step 1. The entries of $\tilde{L}$ are increasing in attention levels. By the Neumann series expansion, $\tilde{L} = \sum_{k=0}^{\infty}(AM)^k$. Since A has non-negative entries and M is a diagonal matrix with non-negative entries, each term $(AM)^k$ has non-negative entries that are weakly increasing in the diagonal elements of M. Therefore, if $\tilde{L}_{ij} > 0$, then $\tilde{L}_{ij}$ is strictly increasing in μ_j.

Step 2. The variance of sector i's marginal cost is increasing in μ_j whenever $\tilde{L}_{ij} > 0$. The diagonal element $V_i^* = [\tilde{L}\Sigma_0\tilde{L}']_{ii} = \sum_{k,l}\tilde{L}_{ik}(\Sigma_0)_{kl}\tilde{L}_{il}$. Since Σ_0 is element-wise non-negative (as $\boldsymbol{\alpha}\boldsymbol{\alpha}'$ and the diagonal matrix Σ both have non-negative entries) and $\tilde{L}$ has non-negative entries, each term in the sum is non-negative. By Step 1, increasing μ_j increases $\tilde{L}_{ij}$, which strictly increases V_i^*.

Step 3. Sector i's optimal attention is increasing in V_i^*. By equation (30), $\mu_i^{BR} = 1 - \omega/(\theta_i\lambda_i V_i^*)$, which is strictly increasing in V_i^*.

Combining Steps 1–3, $\partial\mu_i^{BR}/\partial\mu_j > 0$ whenever $(I - AM)_{ij}^{-1} > 0$. □

C.8. Proof of Proposition A1

PROOF. Differentiating with respect to δ the equilibrium equation for attention in Proposition 1, we have

$$\frac{d\mu_j}{d\delta} = (1 - \mu_j)\left[\frac{1}{\lambda_j}\frac{d\lambda_j}{d\delta} + \frac{1}{V_j^*}\frac{dV_j^*}{d\delta}\right]. \tag{A20}$$

To pin down the total effect of a change in the network structure, we need to understand how δ affects the Domar weight λ_j and the marginal cost variance V_j^*.

Domar weight. Let $A(\delta)$ be the new input-output matrix. Then, $A(\delta) - A = \delta\, e_i\mathbf{1}'$, where $\mathbf{1}'$ is a row vector of 1s and $e_i \in \mathbb{R}^n$ is the i-th standard basis (column) vector. Applying the Sherman–Morrison formula, we have that

$$(I - A(\delta))^{-1} = (I - A)^{-1} + \frac{\delta}{1 - \delta\,\mathbf{1}'(I - A)^{-1}e_i}(I - A)^{-1}e_i \cdot \mathbf{1}'(I - A)^{-1}.$$

Left-multiplying by $\boldsymbol{\gamma}'$ and using $\boldsymbol{\gamma}'(I-A)^{-1}e_i = \boldsymbol{\lambda}'e_i = \lambda_i$ gives

$$\boldsymbol{\lambda}(\delta) = \boldsymbol{\lambda} + \frac{\delta\lambda_i}{1-\delta\,\mathbf{1}'(I-A)^{-1}e_i}\,\mathbf{1}'(I-A)^{-1}.$$

Hence,

$$\left.\frac{d\lambda_j}{d\delta}\right|_{\delta=0} = \lambda_i(\mathbf{1}'(I-A)^{-1})_j \geq 0.$$

Marginal cost volatility. Let $B=(I-A\mathrm{M})^{-1}$ and $B(\delta)=(I-A(\delta)\mathrm{M})^{-1}$. Then, the variance-covariance matrix of the marginal costs is

$$\boldsymbol{\Sigma}_{\boldsymbol{p}^*} = \delta_w^2\sigma_m^2(B(\delta)\boldsymbol{\alpha}(\delta))(B(\delta)\boldsymbol{\alpha}(\delta))' + B(\delta)\Sigma B'(\delta).$$

Because $B(\delta)$ depends on the attention levels, δ affects $\boldsymbol{\Sigma}_{\boldsymbol{p}^*}$ through a direct channel (holding M fixed) and an indirect channel (allowing for δ to affect M).

$$V_j^* = \delta_w^2\sigma_m^2(B(\delta)\boldsymbol{\alpha}(\delta))_j^2 + \sum_{k=1}^n B_{jk}^2(\delta)\sigma_{z_k}^2.$$

$$\text{Direct effect:}\quad \frac{dV_j^*}{d\delta} = 2\left[\delta_w^2\sigma_m^2(B(\delta)\boldsymbol{\alpha}(\delta))_j\frac{d(B(\delta)\boldsymbol{\alpha}(\delta))_j}{d\delta}\right. \\ \left. + \sum_{k=1}^n B_{jk}(\delta)\sigma_{z_k}^2\frac{dB_{jk}(\delta)}{d\delta}\right].$$

From the Sherman–Morrison formula, we have that

$$B(\delta) = B + \frac{\delta}{1-\delta\boldsymbol{\mu}'Be_i}Be_i\boldsymbol{\mu}'B,$$
$$B(\delta)\boldsymbol{\alpha}(\delta) = \left(B + \frac{\delta}{1-\delta\boldsymbol{\mu}'Be_i}Be_iB'\boldsymbol{\mu}\right)(\boldsymbol{\alpha} - n\delta e_i).$$

Hence,

$$\left.\frac{dB(\delta)}{d\delta}\right|_{\delta=0} = Be_i\boldsymbol{\mu}'B \qquad \text{and} \qquad \left.\frac{d(B(\delta)\boldsymbol{\alpha}(\delta))}{d\delta}\right|_{\delta=0} = Be_i(\boldsymbol{\mu}'B\boldsymbol{\alpha} - n),$$

from where we have:

$$\text{Direct effect: } \frac{dV_j^*}{d\delta}\Big|_{\delta=0} = 2B_{ji}\left[\delta_w^2\sigma_m^2(B\boldsymbol{\alpha})_j(\boldsymbol{\mu}'B\boldsymbol{\alpha}-n)+\sum_{k=1}^{n}B_{jk}\boldsymbol{\mu}'B\sigma_{z_k}^2\right] \tag{A21}$$
$$= 2B_{ji}\left[\delta_w^2\sigma_m^2(B\boldsymbol{\alpha})_j(\boldsymbol{\mu}'B\boldsymbol{\alpha}-n)+(B\Sigma B'\boldsymbol{\mu})_j\right]$$

One can write $(B\Sigma B'\boldsymbol{\mu})_j = (\boldsymbol{\Sigma}_{\boldsymbol{p}^*}\boldsymbol{\mu})_j - \delta_w^2\sigma_m^2(B\boldsymbol{\alpha})_j(\boldsymbol{\mu}'B\boldsymbol{\alpha})$. Substituting this into (A21), we have

$$\text{Direct effect: } \quad \frac{dV_j^*}{d\delta}\Big|_{\delta=0} = 2B_{ji}\left[(\boldsymbol{\Sigma}_{\boldsymbol{p}^*}\boldsymbol{\mu})_j - n\delta_w^2\sigma_m^2(B\boldsymbol{\alpha})_j\right].$$

To compute the indirect effect, consider the mapping $\boldsymbol{\mu}=\Phi(\boldsymbol{\mu};A,\boldsymbol{\alpha})$, implied by Proposition 1, such that $\Phi_j = 1-\omega/(\theta\lambda_j V_j^*)$. Differentiating and using $\omega/(\theta\lambda_j) = (1-\Phi_j)V_j^* = (1-\mu_j)V_j^*$ at the fixed point,

$$\frac{d\Phi_j}{d\mu_l} = \frac{2(1-\mu_j)}{V_j^*}\left[\delta_w^2\sigma_m^2(B\boldsymbol{\alpha})_j\frac{d(B\boldsymbol{\alpha})_j}{d\mu_l}+\sum_{k=1}^{n}B_{jk}\sigma_{z_k}^2\frac{dB_{jk}}{d\mu_l}\right].$$

Using $dM/d\mu_l = e_l e_l'$ and $dB/d\mu_l = BAe_le_l'B$, that is, $dB_{jk}/d\mu_l = (BA)_{jl}B_{lk}$, we have

$$\frac{d\Phi_j}{d\mu_l} = \frac{2(1-\mu_j)(BA)_{jl}}{V_j^*}\left[\delta_w^2\sigma_m^2(B\boldsymbol{\alpha})_j(B\boldsymbol{\alpha})_l+\underbrace{\sum_{k=1}^{n}B_{jk}B_{lk}\sigma_{z_k}^2}_{=\boldsymbol{\Sigma}_{\boldsymbol{p}^*}(jl)-\delta_w^2\sigma_m^2(B\boldsymbol{\alpha})_j(B\boldsymbol{\alpha})_l}\right]$$
$$= \frac{2(1-\mu_j)(BA)_{jl}\boldsymbol{\Sigma}_{\boldsymbol{p}^*}(jl)}{V_j^*}.$$

The total effect of δ on μ_j is given by:

$$\frac{d\mu_j}{d\delta} = (1-\mu_j)\left[\frac{1}{\lambda_j}\frac{d\lambda_j}{d\delta}+\frac{1}{V_j^*}\frac{dV_j^*}{d\delta}\right]+\sum_l\underbrace{\frac{d\Phi_j}{d\mu_l}}_{[D_\mu\Phi]_{jl}}\frac{d\mu_l}{d\delta}.$$

Let $\boldsymbol{g}_j = (1-\mu_j)\left[\frac{1}{\lambda_j}\frac{d\lambda_j}{d\delta} + \frac{1}{V_j^*}\frac{dV_j^*}{d\delta}\right]$. Then, the total effect of δ on μ_j is given by:

$$\frac{d\boldsymbol{\mu}}{d\delta} = (I - D_\mu \Phi)^{-1}\boldsymbol{g}.$$

We are interested in unique solutions; hence, we only consider parameterizations for which the spectral radius of $D_\mu\Phi$ is less than 1, which in turn guarantees uniqueness (see also Remark 1 in the main text). Combined with Lemma A1, which ensures that every entry of $D_\mu\Phi$ is nonnegative, this condition implies that $D = (I - D_\mu\Phi)^{-1}$ has positive elements. □

C.9. Proof of Lemma A2

PROOF. We proceed in three steps.

First, we decompose the CPI into an own-sector and rest-of-economy component. Integrating out firm-specific noise in (26) yields industry-level prices $p_{it} = \mu_i p_{it}^*$, so

$$p_t^{CPI} = \boldsymbol{\gamma}'\boldsymbol{p}_t = \sum_j \gamma_j \mu_j p_{jt}^* = \gamma_i \mu_i p_{it}^* + \underbrace{\textstyle\sum_{j\neq i}\gamma_j\mu_j p_{jt}^*}_{\equiv S_{-i,t}}.$$

The comparative static increases γ_i while scaling other consumption shares proportionally, so that $\sum_j \gamma_j = 1$ is maintained. Under this scaling, each γ_j for $j \neq i$ changes proportionally to $(1-\gamma_i)$, and hence $S_{-i,t} = (1-\gamma_i)\sum_{j\neq i}\frac{\gamma_j}{1-\gamma_i}\mu_j p_{jt}^*$ is invariant to γ_i, since the ratios $\gamma_j/(1-\gamma_i)$ are held constant.

Second, we express R_i^2 in terms of γ_i. Let $V_i^* \equiv Var(p_{it}^*)$ as in (30), $C_S \equiv Cov(p_{it}^*, S_{-i,t})$, and $V_S \equiv Var(S_{-i,t})$. All three are invariant to γ_i, since the joint distribution of industry marginal costs depends on attention levels and the production structure $(A, \boldsymbol{\alpha})$, not on consumption shares. By bilinearity of covariance,

$$\begin{aligned} Cov(p_{it}^*, p_t^{CPI}) &= \gamma_i \mu_i V_i^* + C_S, \\ Var(p_t^{CPI}) &= \gamma_i^2 \mu_i^2 V_i^* + 2\gamma_i\mu_i C_S + V_S, \end{aligned}$$

so that $R_i^2 = \left[Cov(p_{it}^*, p_t^{CPI})\right]^2 / \left[V_i^* \cdot Var(p_t^{CPI})\right]$.

Third, we differentiate R_i^2 with respect to γ_i. Since V_i^*, C_S, and V_S do not depend on γ_i, differentiating and simplifying yields

$$\frac{\partial R_i^2}{\partial \gamma_i} = \frac{2\mu_i \left(V_i^* V_S - C_S^2\right)}{V_i^* \left[Var(p_t^{CPI})\right]^2} \cdot Cov(p_{it}^*, p_t^{CPI}).$$

Since $Corr(p_{it}^*, S_{-i,t})^2 \leq 1$, we have $V_i^* V_S - C_S^2 \geq 0$. The covariance condition $Cov(p_{it}^*, p_t^{CPI}) \geq 0$ is guaranteed in our model: $Cov(p_{it}^*, p_t^{CPI}) = \sum_j \gamma_j \mu_j Cov(p_{it}^*, p_{jt}^*)$, where each $Cov(p_{it}^*, p_{jt}^*) = \boldsymbol{h}_i' \Sigma_x \boldsymbol{h}_j \geq 0$ since the rows $\boldsymbol{h}_i$ of $(I - AM)^{-1}$ have non-negative entries and $\Sigma_x \equiv \boldsymbol{\alpha\alpha}' \delta_w^2 \sigma_m^2 + \Sigma$ is element-wise non-negative. Since $\mu_i > 0$, it follows that $\partial R_i^2 / \partial \gamma_i \geq 0$. The inequality is strict whenever $Corr(p_{it}^*, S_{-i,t})^2 < 1$ and $Cov(p_{it}^*, p_t^{CPI}) > 0$. □

C.10. Proof of Lemma A3 and Corollary A1

PROOF. We proceed in four steps.

Step 1: closed-form prices. Under constant returns to scale, the pricing equation $p_{it}^* = \alpha_i \delta_w m_t + \bar{\mu} \sum_j a_{ij} p_{jt}^* - z_{it}$ gives, after solving the 2×2 system,

$$p_{1t}^* = D\big[M_1 \delta_w m_t - z_{1t} - \bar{\mu} b z_{2t}\big], \qquad p_{2t}^* = D\big[M_2 \delta_w m_t - \bar{\mu} a z_{1t} - z_{2t}\big],$$

where $D \equiv 1/(1 - \bar{\mu}^2 ab)$, $M_1 \equiv (1-b) + \bar{\mu} b (1-a)$, and $M_2 \equiv (1-a) + \bar{\mu} a (1-b)$. Direct computation gives

$$M_1 - M_2 = (a-b)(1-\bar{\mu}), \qquad M_1 + M_2 = 2 - a - b + \bar{\mu}(a + b - 2ab).$$

Step 2: scaffolding identity. Under $\gamma_1 = \gamma_2 = 1/2$ and $\mu_1 = \mu_2 = \bar{\mu}$, $p_t^{CPI} = (\bar{\mu}/2)(p_{1t}^* + p_{2t}^*)$, so the scalar $\bar{\mu}/2$ drops out of R_i^2 and

$$R_i^2 = \frac{(V_i^* + C_{12})^2}{V_i^* \cdot (V_1^* + V_2^* + 2C_{12})},$$

where $V_i^* \equiv Var(p_{it}^*)$ and $C_{12} \equiv Cov(p_{1t}^*, p_{2t}^*)$. Algebraic manipulation yields

$$R_1^2 - R_2^2 = \frac{(V_1^* - V_2^*)(V_1^* V_2^* - C_{12}^2)}{V_1^* V_2^* \cdot (V_1^* + V_2^* + 2C_{12})}.$$

Cauchy–Schwarz implies $V_1^* V_2^* - C_{12}^2 \geq 0$, with strict inequality whenever p_{1t}^* and p_{2t}^* are not perfectly correlated (generic under $\sigma_z^2 > 0$). The denominator is strictly positive, so

$\text{sign}(R_1^2 - R_2^2) = \text{sign}(V_1^* - V_2^*)$.

Step 3: threshold. From the closed-form prices,

$$V_1^*/D^2 = M_1^2\delta_w^2\sigma_m^2 + \sigma_{z,1}^2 + \bar{\mu}^2 b^2\sigma_{z,2}^2,$$
$$V_2^*/D^2 = M_2^2\delta_w^2\sigma_m^2 + \bar{\mu}^2 a^2\sigma_{z,1}^2 + \sigma_{z,2}^2.$$

Imposing $\sigma_{z,1}^2 = \sigma_{z,2}^2 = \sigma_z^2$ and using $M_1^2 - M_2^2 = (M_1 - M_2)(M_1 + M_2)$ from Step 1,

$$(V_1^* - V_2^*)/D^2 = (a-b)(1-\bar{\mu})\big[2 - a - b + \bar{\mu}(a + b - 2ab)\big]\delta_w^2\sigma_m^2 - \bar{\mu}^2(a^2 - b^2)\sigma_z^2.$$

Under $a > b$, dividing by $(a-b)D^2 > 0$ gives $V_1^* > V_2^* \iff \sigma_m^2/\sigma_z^2 > T(a,b)$, with $T(a,b)$ as in (A3). Combined with Step 2, this establishes Lemma A3.

Step 4: comparative statics. Write $T(a,b) = N(a,b)/[(1-\bar{\mu})B(a,b)\delta_w^2]$ with $N(a,b) \equiv \bar{\mu}^2(a+b)$ and $B(a,b) \equiv 2 - (a+b)(1-\bar{\mu}) - 2\bar{\mu}ab$. Differentiation gives $\partial_a N = \partial_b N = \bar{\mu}^2 > 0$, $\partial_a B = -(1 - \bar{\mu} + 2\bar{\mu}b) < 0$, and $\partial_b B = -(1 - \bar{\mu} + 2\bar{\mu}a) < 0$. Hence $\partial T/\partial a > 0$ and $\partial T/\partial b > 0$, establishing parts (i) and (ii) of Corollary A1. For part (iii), set $da = -db > 0$ at fixed $a + b$: then $dN = \bar{\mu}^2(da + db) = 0$ and $dB = -2\bar{\mu}(b\,da + a\,db) = 2\bar{\mu}(a-b)\,da > 0$ under $a > b$, so T strictly falls. $\square$