Online Appendix for “ Expectations and Frictions: Lessons from a Quantitative Model with Dispersed Information”

Derivation of the Model

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A Derivation and log-linearization of households’ problem

A.1 Euler Equation

We first detrend and log-linearize equations (34) and (35) in order to get the usual Euler equation for consumption.

Detrending equation (34) we have

$$E_{ht}\left[\gamma^t \Lambda_{h,t} P_t\right] = \frac{E_{ht}\left[e^{\eta_t^c}\right]}{C_{h,t}/\gamma^t - \varphi C_{h,t-1}/(\gamma \cdot \gamma^{t-1})}$$

$$\Rightarrow \quad E_{ht}[\hat{\Lambda}_{h,t}] = \frac{E_{ht}\left[e^{\eta_t^c}\right]}{\hat{C}_{h,t} - \varphi/\gamma \hat{C}_{h,t-1}}$$

We log-linearize the equation above

$$\bar{\Lambda}_h(1 + E_{ht}[\lambda_{h,t}] = 1 + \frac{E_{ht}[eta_t^c]}{\bar{C}_h[1 - \varphi/\gamma + c_{h,t} - \varphi/\gamma c_{h,t-1}]}$$

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Using the steady-state result $\bar{\Lambda}_h = \left[\bar{C}_h \left(1 - \varphi/\gamma\right)\right]^{-1}$ we find

$$(1 + E_{ht}[\lambda_{h,t}]\left(1 - \frac{\varphi}{\gamma} + c_{h,t} - \frac{\varphi}{\gamma}c_{h,t-1}\right) = \left(1 - \frac{\varphi}{\gamma}\right)(1 + E_{ht}\left[\eta_t^c\right])$$

Rearranging terms

$$E_{ht}[\lambda_{h,t}] = \frac{\varphi/\gamma}{1-\varphi/\gamma}c_{h,t-1} - \frac{1}{1-\varphi/\gamma}c_{h,t} + E_{ht}\left[\eta_t^c\right] \tag{A.1}$$

The detrended version of (35) is

$$\begin{aligned} E_{ht}[\gamma^t \Lambda_{h,t} P_t] &= \frac{\beta}{\gamma} E_{ht}\left[\gamma^{t+1}\Lambda_{h,t+1}P_{t+1}\frac{P_t}{P_{t+1}}R_t\right] \\ \Rightarrow \quad E_{ht}[\hat{\Lambda}_{h,t}] &= \frac{\beta}{\gamma} E_{ht}\left[\hat{\Lambda}_{h,t+1}\frac{R_t}{\Pi_{t+1}}\right] \end{aligned}$$

We now log-linearize the equation above

$$\bar{\Lambda}_h(1 + E_{ht}[\lambda_{h,t}]) = \frac{\beta}{\gamma}\frac{\bar{R}}{\bar{\Pi}}\bar{\Lambda}_h E_{ht}\left[1 + \lambda_{h,t+1} + r_t - \pi_{t+1}\right]$$

Using the steady-state result that $\bar{\Pi}/\bar{R} = \beta/\gamma$ and rearranging terms

$$E_{ht}[\lambda_{h,t}] = E_{ht}[\lambda_{h,t+1} + r_t - \pi_{t+1}] \tag{A.2}$$

Substituting (A.1) into (A.2) implies

$$\begin{aligned} &\frac{\varphi/\gamma}{1-\varphi/\gamma}c_{h,t-1} - \frac{1}{1-\varphi/\gamma}c_{h,t} + \pi_t - E_{ht}[\pi_t] + E_{ht}\left[\eta_t^c\right] = \pi_t - E_{ht}[\pi_t] + \\ &E_{ht}\left[\frac{\varphi/\gamma}{1-\varphi/\gamma}c_{h,t} - \frac{1}{1-\varphi/\gamma}c_{h,t+1} + \pi_{t+1} - E_{ht}[\pi_{t+1}] + E_{ht}\left[\eta_{t+1}^c\right] + r_t - \pi_{t+1}\right] \end{aligned}$$

Rearranging terms

$$c_{h,t} = \frac{\varphi/\gamma}{1+\varphi/\gamma}c_{h,t-1} + \frac{1}{1+\varphi/\gamma}E_{ht}[c_{h,t+1}] - \frac{1-\varphi/\gamma}{1+\varphi/\gamma}E_{ht}[r_t - \pi_{t+1}] - \frac{1-\varphi/\gamma}{1+\varphi/\gamma}E_{ht}\left[\eta_{t+1}^c - \eta_t^c\right]$$

Using the result $c_{h,t} = c_t$ for all h and integrating over h, we get

$$c_t = \frac{\varphi/\gamma}{1+\varphi/\gamma} c_{t-1} + \frac{1}{1+\varphi/\gamma} E_t^{(1)}[c_{t+1}] - \frac{1-\varphi/\gamma}{1+\varphi/\gamma} E_t^{(1)}[r_t - \pi_{t+1}] - \frac{1-\varphi/\gamma}{1+\varphi/\gamma} E_t^{(1)} \left[\eta_{t+1}^c - \eta_t^c\right] \quad \text{(A.3)}$$

A.2 Equilibrium condition for investment

We use equations (37) and (35) to get an equilibrium condition for investment.

Dividing each term of equation (37) by $\Lambda_{h,t} P_t$ and using the definition $Q_{h,t} \equiv \frac{\Phi_{h,t}}{\Lambda_{h,t} P_t}$

$$\begin{aligned}\frac{E_{ht}[P_t/P_{t-1}]}{P_t/P_{t-1}} =& E_{ht}\left[Q_{h,t} e^{\eta_t^i}\right] \left(1 - S\left(\frac{I_{h,t}}{I_{h,t-1}}\right) - S'\left(\frac{I_{h,t}}{I_{h,t-1}}\right) \frac{I_{h,t}}{I_{h,t-1}}\right) + \\ & \frac{\beta}{\Lambda_{h,t} P_t} E_{ht}\left[Q_{h,t+1} \Lambda_{h,t+1} P_{t+1} e^{\eta_{t+1}^i} S'\left(\frac{I_{h,t+1}}{I_{h,t}}\right) \left(\frac{I_{h,t+1}}{I_{h,t}}\right)^2\right]\end{aligned}$$

We now detrend the variables to get

$$\begin{aligned}\frac{E_{ht}[\Pi_t]}{\Pi_t} =& E_{ht}\left[Q_{h,t} e^{\eta_t^i}\right] \left(1 - S\left(\frac{\gamma \hat{I}_{h,t}}{\hat{I}_{h,t-1}}\right) - S'\left(\frac{\gamma \hat{I}_{h,t}}{\hat{I}_{h,t-1}}\right) \frac{\gamma \hat{I}_{h,t}}{\hat{I}_{h,t-1}}\right) + \\ & \frac{\beta}{\gamma \hat{\Lambda}_{h,t}} E_{ht}\left[Q_{h,t+1} \hat{\Lambda}_{h,t+1} e^{\eta_{t+1}^i} S'\left(\frac{\gamma \hat{I}_{h,t+1}}{I_{h,t}}\right) \left(\frac{\gamma \hat{I}_{h,t+1}}{\hat{I}_{h,t}}\right)^2\right]\end{aligned} \quad \text{(A.4)}$$

We now log-linearize the equation using the following approximations

$$\begin{aligned}\frac{\hat{I}_{h,t}}{\hat{I}_{h,t-1}} &\approx 1 + i_{h,t} - i_{h,t-1} \\ S\left(\gamma(1 + i_{h,t} - i_{h,t-1})\right) &\approx S(\gamma) + \gamma S'(\gamma)\left(i_{h,t} - i_{h,t-1}\right) = 0 \\ S'\left(\gamma(1 + i_{h,t+1} - i_{h,t}\right) &\approx S'(\gamma) + \gamma S''(\gamma)(i_{h,t+1} - i_{h,t}) = \gamma s''(i_{h,t+1} - i_{h,t})\end{aligned}$$

We will have

$$\begin{aligned}1 + E_{ht}[\pi_t] - \pi_t = & \bar{Q}_h \left(1 + q_{h,t} + E_{ht}\left[\eta_t^i\right]\right) \left(1 - \gamma^2 s''(i_{h,t} - i_{h,t-1})(1 + i_{h,t} - i_{h,t-1})\right) + \\ & \bar{Q}_h \beta \gamma^2 (1 - \lambda_{h,t}) E_{ht}\left[(1 + \lambda_{h,t+1})(1 + \eta_{t+1}^i) s''(i_{h,t+1} - i_{h,t})(1 + i_{h,t} - i_{h,t-1})^2\right]\end{aligned}$$

We arrive to

$$E_{ht}[\pi_t] - \pi_t = q_{h,t} + E_{ht}[\eta_t^i] - \gamma^2 s''(i_{h,t} - i_{h,t-1}) + \beta\gamma^2 E_{ht}\left[s''(i_{h,t+1} - i_{h,t})\right]$$

Rearranging terms, we get

$$i_{h,t} = \left(\frac{1}{1+\beta}\right) i_{h,t-1} + \left(\frac{\beta}{1+\beta}\right) E_{ht}[i_{h,t+1}] + \left(\frac{1}{s''(1+\beta)\gamma^2}\right)\left(q_{h,t} + E_{ht}\left[\eta^i_t\right] + \pi_t - E_{ht}[\pi_t]\right)$$

Using the result $i_{h,t} = i_t$ and $q_{h,t} = q_t$ for all h and integrating over h, we have

$$i_t = \left(\frac{1}{1+\beta}\right) i_{t-1} + \left(\frac{\beta}{1+\beta}\right) E^{(1)}_t[i_{t+1}] + \left(\frac{1}{s''(1+\beta)\gamma^2}\right)\left(E^{(1)}_t\left[q_t + \eta^i_t\right] + \pi_t - E^{(1)}_t[\pi_t]\right) \quad \text{(A.5)}$$

A.3 Asset pricing equation for capital stock

We start detrending equation (36) by dividing each term by $\Lambda_{h,t}P_t$ and using the definition of Q_t

$$Q_{h,t} = \frac{\beta}{\gamma\hat{\Lambda}_{h,t}} E_{ht}\left[\hat{\Lambda}_{h,t+1}\left(\hat{R}^k_{t+1}U_{h,t+1} - a(U_{h,t+1})\right)\right] + (1-\delta)\frac{\beta}{\gamma\hat{\Lambda}_{h,t}} E_{ht}\left[Q_{h,t+1}\hat{\Lambda}_{h,t+1}\right]$$

We start log-linearizing using the following approximation

$$a(U_{h,t+1}) \approx a(\bar{U}_h) + a'(\bar{U}_h)(U_{h,t+1} - \bar{U}_h) = \bar{R}^k(U_{h,t+1} - 1) \approx \bar{R}^k u_{h,t+1}$$

We now use the following steady-state results to simplify the expression: $\bar{Q} = 1$, $\bar{U} = 1$ and $\bar{R}^k = a'(\bar{U})$

$$1 + q_{h,t} = \left(1 - \frac{(1-\delta)\beta}{\gamma}\right)(1-\lambda_t)\left(1 + E_{ht}\left[\lambda_{h,t+1} + r^k_{t+1}\right]\right) + \left(\frac{(1-\delta)\beta}{\gamma}\right)(1-\lambda_{h,t})\left(1 + E_{ht}\left[q_{h,t+1} + \lambda_{h,t+1}\right]\right)$$

Simplifying further gives us

$$q_{h,t} = \left(1 - \frac{(1-\delta)\beta}{\gamma}\right) E_{ht}\left[r^k_{t+1}\right] + \left(\frac{(1-\delta)\beta}{\gamma}\right) E_{ht}[q_{h,t+1}] + E_{ht}[\lambda_{h,t+1}] - \lambda_{h,t} \quad \text{(A.6)}$$

We may rewrite (A.2) as

$$E_{ht}[\lambda_{h,t+1}] - \lambda_{h,t} = E_{ht}[\pi_t] - \pi_t - E_{ht}[r_t - \pi_{t+1}]$$

Plugging the result above into (A.6) leads to

$$q_{h,t} = \left(\frac{\beta(1-\delta)}{\gamma}\right) E_{ht}[q_{h,t+1}] + \left(1 - \frac{\beta(1-\delta)}{\gamma}\right) E_{ht}\left[r^k_{t+1}\right] - E_{ht}[r_t - \pi_{t+1}] - (\pi_t - E_{ht}[\pi_t])$$

Integrating over h, we finally get

$$q_t = \left(\frac{\beta(1-\delta)}{\gamma}\right) \int_0^1 E_{ht}[q_{h,t+1}]dh + \left(1 - \frac{\beta(1-\delta)}{\gamma}\right) E^{(1)}_t\left[r^k_{t+1}\right] - E^{(1)}_t[r_t - \pi_{t+1}] - \left(\pi_t - E^{(1)}_t[\pi_t]\right) \tag{A.7}$$

If we use the result that $q_{h,t} = q_t$ for all h, we have instead

$$q_t = \left(\frac{\beta(1-\delta)}{\gamma}\right) E^{(1)}_t[q_{t+1}] + \left(1 - \frac{\beta(1-\delta)}{\gamma}\right) E^{(1)}_t[r^k_{t+1}] - E^{(1)}_t[r_t - \pi_{t+1}] - (\pi_t - E^{(1)}_t[\pi_t])$$

A.4 Law of motion of capital

We first detrend equation (17) to get a log-linear version of the law of motion of capital.

$$K_{h,t}/\gamma^t = (1-\delta)K_{h,t-1}/(\gamma\cdot\gamma^{t-1}) + e^{\eta^I_t}\left(1 - S(I_{h,t}/\gamma^t/I_{h,t-1}/(\gamma\cdot\gamma^{t-1}))\right) I_{h,t}/\gamma^t$$
$$\Rightarrow\ \hat{K}_{h,t} = \left(\frac{1-\delta}{\gamma}\right)\hat{K}_{h,t-1} + e^{\eta^I_t}\left(1 - S\left(\gamma\frac{\hat{I}_{h,t}}{\hat{I}_{h,t-1}})\right)\right)\hat{I}_{h,t}$$

We now log-linearize the equation

$$\bar{K}_h(1+k_{h,t}) = \left(\frac{1-\delta}{\gamma}\right)\bar{K}_h(1+k_{h,t-1}) + (1+\eta^i_t)\left(1 - S\left(\gamma(1+i_{h,t}-i_{h,t-1})\right)\right)\bar{I}_h(1+i_{h,t})$$

Using the steady-state result $\bar{I}_h = (1-(1-\delta)/\gamma)\bar{K}_h$

$$1 + k_{h,t} = \left(\frac{1-\delta}{\gamma}\right)(1+k_{h,t-1}) + (1+\eta^i_t)\left(1 - S\left(\gamma(1+i_{h,t}-i_{h,t-1})\right)\right)\left(1 - \frac{1-\delta}{\gamma}\right)(1+i_{h,t})$$
$$\Rightarrow\ k_{h,t} = \left(\frac{1-\delta}{\gamma}\right)k_{h,t-1} + \left(1 - \frac{1-\delta}{\gamma}\right)\left(i_{h,t} + \eta^i_t - S\left(\gamma(1+i_{h,t}-i_{h,t-1})\right)\left(1 + i_{h,t} + \eta^i_t\right)\right)$$

We use the following approximation

$$S\left(\gamma(1+i_{h,t}-i_{h,t-1})\right) \approx S(\gamma)+\gamma S'(\gamma)\left(i_{h,t}-i_{h,t-1}\right)=0$$

We find that

$$k_{h,t}=\left(\frac{1-\delta}{\gamma}\right)k_{h,t-1}+\left(1-\frac{1-\delta}{\gamma}\right)\left(i_{h,t}+\eta_t^i\right)$$

Integrating over h

$$k_t=\left(\frac{1-\delta}{\gamma}\right)k_{t-1}+\left(1-\frac{1-\delta}{\gamma}\right)\left(i_t+\eta_t^i\right) \tag{A.8}$$

A.5 Capital utilization level

Based on equation (38), we obtain a log-linearized of the capital utilization function by first dividing both sides of the equation by P_t

$$a'(U_{h,t})E_{ht}\left[\frac{P_t}{P_{t-1}}\right]=E_{ht}\left[\hat{R}_t^k\frac{P_t}{P_{t-1}}\right]$$

We now log-linearize each term and use the following approximation

$$a'(U_{h,t})\approx a'(\bar{U}_h)+a''(\bar{U}_h)(U_{h,t}-\bar{U}_h)=\bar{R}^k+a''u_{h,t}$$

This leads to

$$\begin{aligned}&\bar{\Pi}(\bar{R}^k+a''u_{h,t}+\bar{R}^kE_{ht}[\pi_t])=\bar{R}^k\bar{\Pi}(1+E_{ht}[r_t^k+\pi_t])\\ \Rightarrow\ & u_{h,t}=\left(\bar{R}^k/a''\right)E_{ht}[r_t^k]\end{aligned}$$

Integrating over h, we have

$$u_t=(\bar{R}^k/a'')E_t^{(1)}[r_t^k] \tag{A.9}$$

A.6 Utilized capital level

Equation (19) becomes

$$
\begin{aligned}
& K^u_{h,t}/\gamma^t = U_{h,t}K_{h,t-1}/(\gamma\cdot\gamma^t) \\
\Rightarrow\quad & \hat{K}^u_{h,t} = U_{h,t}\hat{K}_{h,t-1}/\gamma \\
\Rightarrow\quad & k^u_{h,t} = u_{h,t} + k_{h,t-1}
\end{aligned}
$$

Integrating over $h \in [0,1]$, we get

$$
k^u_t = u_t + k_{t-1} \tag{A.10}
$$

A.7 Marginal cost of labor

From the definition of marginal cost of labor (40)

$$
\begin{aligned}
& MRS_{h,t}/\gamma^t = L^{\chi}_{h,t}\left(C_{h,t}/\gamma^t - \varphi C_{h,t-1}/(\gamma\cdot\gamma^{t-1}\right) \\
& \widehat{MRS}_{h,t} = L^{\chi}_{h,t}\left(\hat{C}_{h,t} - \varphi/\gamma\hat{C}_{h,t-1}\right)
\end{aligned}
$$

We now log-linearize the variables and use the approximation $(1+l_{h,t})^{\chi} \approx 1 + \chi l_{h,t}$

$$
M\bar{R}S(1+mrs_{h,t}) = \bar{L}^{\chi}\bar{C}(1+\chi l_{h,t})(1 + c_{h,t} - \varphi/\gamma - \varphi/\gamma c_{h,t-1})
$$

Using the fact that $M\bar{R}S = \bar{C}(1-\varphi/\gamma)\bar{L}^{\chi}$, we get

$$
(1+mrs_{h,t})\left(1-\frac{\varphi}{\gamma}\right) = 1 - \frac{\varphi}{\gamma} + \left(1-\frac{\varphi}{\gamma}\right)\chi l_{h,t} + c_{h,t} - \frac{\varphi}{\gamma}c_{h,t-1}
$$

We rearrange terms to get

$$
mrs_{h,t} = \left(\frac{1}{1-\varphi/\gamma}\right)\left(c_{h,t} - \frac{\varphi}{\gamma}c_{h,t-1}\right) + \chi l_{h,t}
$$

We finally integrate over h to get

$$
mrs_t = \left(\frac{1}{1-\varphi/\gamma}\right)(c_t - \varphi/\gamma c_{t-1}) + \chi l_t \tag{A.11}
$$

A.8 Resource constraint

The resource constrained is defined by equation (29) as follows

$$Y_t = C_t + I_t + a(U_t)K_t + G_t$$

Log-linearizing each term leads to

$$\bar{Y}(1 + y_t) = \bar{C}(1 + c_t) + \bar{I}(1 + i_t) + \bar{K}\bar{R}^k u_t + \bar{G}(1 + g_t)$$

We use the steady-state result that $\bar{Y} = \bar{C} + \bar{I} + \bar{G}$ and divide each side by $\bar{Y}$ to simplify the expression above to

$$y_t = c_y c_t + i_y i_t + \bar{R}^k k_y u_t + g_y g_t,$$

where $c_y = \bar{C}/\bar{Y}$, $i_y = \bar{I}/\bar{Y}$, $K_y = \bar{K}/\bar{Y}$ and $g_y = \bar{G}/\bar{Y}$. We now derive a formula for g_t by noticing that government expenditures follow the rule below

$$\begin{aligned} &\frac{G_t}{Y_t} = g_y + \eta_t^g \Rightarrow g_y(1 + g_t - y_t) = g_y + \eta_t^g \Rightarrow \\ &g_t = y_t + \frac{\eta_t^g}{g_y} \end{aligned}$$

Plugging the result above into the log-linearized version of the resource constraint implies

$$y_t = \frac{1}{1 - g_y}\left(\frac{\bar{C}}{\bar{Y}}c_t + \frac{\bar{I}}{\bar{Y}}i_t + \frac{\bar{R}^k\bar{K}}{\bar{Y}}u_t + \eta_t^g\right) \tag{A.12}$$

B Derivation of firms' problem

B.1 Cost minimization problem

$$\begin{aligned} &\min_{\{K_{i,t}, L_{i,t}\}} E_{it}\left[W_t L_{i,t} + R_t^k K_{i,t}\right] - \gamma^t \Phi_p \\ &\text{s.t. } Y_{i,t} = A_t K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}, \end{aligned}$$

where A_t follows

$$\log(A_t) = \rho_a \log(A_{t-1}) + \varepsilon_t^a \tag{B.1}$$

Lagrangean:

$$\min_{\{K_{i,t},L_{i,t}\}} \mathcal{L} = E_{it}\left[W_t L_{i,t} + R_t^k K_{i,t} - \gamma^t \Phi_p + \mu_{i,t}\left[Y_{i,t} - A_t K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}\right]\right]$$

FOC:

$$E_{it}\left[W_t\right] = (1-\alpha)\mu_{i,t}\frac{E_{it}\left[Y_{i,t}\right]}{L_{i,t}}$$
$$E_{it}\left[R_t^k\right] = \alpha\mu_{i,t}\frac{E_{it}\left[Y_{i,t}\right]}{K_{i,t}}$$

Dividing one FOC by another:

$$\frac{E_{it}\left[W_t\right]}{E_{it}\left[R_t^k\right]} = \frac{1-\alpha}{\alpha}\frac{K_{i,t}}{L_{i,t}} \tag{B.2}$$

substituting into the production function and solving for $K_{i,t}$ and Li,t as a function of $Y_{i,t}$:

$$Y_{i,t} = A_t\left[\frac{\alpha}{1-\alpha}\frac{E_{it}\left[W_t\right]}{E_{it}\left[R_t^k\right]}\right]^{\alpha} L_{i,t} \Rightarrow \qquad L_{i,t} = \left[\frac{1-\alpha}{\alpha}\frac{E_{it}\left[R_t^k\right]}{E_{it}\left[W_t\right]}\right]^{\alpha}\frac{Y_{i,t}}{A_t}$$
$$Y_{i,t} = A_t\left[\frac{1-\alpha}{\alpha}\frac{E_{it}\left[R_t^k\right]}{E_{it}\left[W_t\right]}\right]^{1-\alpha} K_{i,t} \Rightarrow \qquad K_{i,t} = \left[\frac{\alpha}{1-\alpha}\frac{E_{it}\left[W_t\right]}{E_{it}\left[R_t^k\right]}\right]^{1-\alpha}\frac{Y_{i,t}}{A_t}$$

Substituting both into the total cost:

$$\begin{aligned}\mathcal{C}(Y_{i,t}) &= \left[W_t\left[\frac{1-\alpha}{\alpha}\frac{E_{it}\left[R_t^k\right]}{E_{it}\left[W_t\right]}\right]^{\alpha} + R_t^k\left[\frac{\alpha}{1-\alpha}\frac{E_{it}\left[W_t\right]}{E_{it}\left[R_t^k\right]}\right]^{1-\alpha}\right]\frac{Y_{i,t}}{A_t} - A_t\Phi_p \\ &= \left[\frac{(1-\alpha)W_t E_{it}\left[R_t^k\right] + \alpha R_t^k E_{it}\left[W_t\right]}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}E_{it}\left[W_t\right]^{\alpha} E_{it}\left[R_t^k\right]^{1-\alpha}}\right]\frac{Y_{i,t}}{A_t} - A_t\Phi_p\end{aligned}$$

Marginal cost:

$$MC_{i,t} \equiv \mathcal{C}'(Y_{i,t}) = \left[\frac{(1-\alpha)W_t E_{it}\left[R_t^k\right] + \alpha R_t^k E_{it}\left[W_t\right]}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}E_{it}\left[W_t\right]^{\alpha} E_{it}\left[R_t^k\right]^{1-\alpha}}\right]\frac{1}{A_t} \tag{B.3}$$

B.2 Price index

The price index is given by

$$P_t = \left[\int_0^1 P_{i,t}^{-\frac{1}{\mu_t^p}} di \right]^{-\mu_t^p}$$

The firms that do not optimize, follow the indexation rule given by

$$X_t = \Pi_{t-1}^{\iota_p} \bar{\Pi}^{1-\iota_p}$$

where $\iota_p \in [0, 1]$.

Observe that

$$\begin{aligned} P_t^{-\frac{1}{\mu_t^p}} &= \int_0^1 P_{i,t}^{-\frac{1}{\mu_t^p}} di = \int_0^{\xi_p} (X_t P_{i,t-1})^{-\frac{1}{\mu_t^p}} di + \int_{\xi_p}^1 (P_{i,t}^*)^{-\frac{1}{\mu_t^p}} di \\ &= \xi_p X_t^{-\frac{1}{\mu_t^p}} \int_0^1 P_{i,t-1}^{-\frac{1}{\mu_t^p}} di + (1-\xi_p) \int_0^1 (P_{i,t}^*)^{-\frac{1}{\mu_t^p}} di \\ &= \xi_p (X_t P_{t-1})^{-\frac{1}{\mu_t^p}} + (1-\xi_p)(P_t^*)^{-\frac{1}{\mu_t^p}} \end{aligned}$$

where the last equality uses the definition of P_t in period $t-1$ and uses the definition of aggregate optimal price given by $P_t^* = \left[\int_0^1 (P_{i,t}^*)^{-\frac{1}{\mu_t^p}} di \right]^{-\mu_t^p}$.

Dividing both sides by $P_{t-1}^{-\frac{1}{\mu_t^p}}$ and using the definition of Π_t and X_t:

$$\Pi_t^{-\frac{1}{\mu_t^p}} = \xi_p \left(\Pi_{t-1}^{\iota_p} \bar{\Pi}^{1-\iota_p} \right)^{-\frac{1}{\mu_t^p}} + (1-\xi_p) \left(\hat{P}_t^* \right)^{-\frac{1}{\mu_t^p}} \tag{B.4}$$

where $\hat{P}_t^* = \frac{P_t^*}{P_{t-1}}$.

B.3 Optimal price

$$\max_{\{P^*_{i,t}\}} \quad \sum_{s=0}^{\infty} \xi_p^s E_{it} \left[\Lambda_{t,t+s}\left(P^*_{i,t} X_{t,t+s} Y_{t+s|t} - \mathcal{C}(Y_{t+s|t})\right)\right]$$

$$\text{s.t.} \quad Y_{t+s|t} = \left(\frac{P^*_{i,t} X_{t,t+s}}{P_{t+s}}\right)^{-\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}} C_{t+s}$$

$$X_{t,t+s} = \begin{cases} \prod_{j=1}^{s}(\Pi_{t+j-1}^{\iota_p}\bar{\Pi}^{1-\iota_p}), & \text{for } s \geq 1 \\ 1, & \text{for } s = 0 \end{cases}$$

$$\Lambda_{t,t+s} = \beta^s \frac{\lambda_{t+s}}{\lambda_t} = \beta^s \frac{u_c(C_{t+s})}{u_c(C_t)} \frac{P_t}{P_{t+s}}$$

where $Y_{t+s|t}$ denote the output from period $t+s$ given that the last price update was done in period t.

FOC:

$$\sum_{s=0}^{\infty} \xi_p^s E_{it} \left[\Lambda_{t,t+s}\left(X_{t,t+s} Y_{t+s|t} - \left(\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}\right) P^*_{i,t} X_{t,t+s} \frac{Y_{t+s|t}}{P^*_{i,t}} + \mathcal{C}'(Y_{t+s|t}) \left(\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}\right) \frac{Y_{t+s|t}}{P^*_{i,t}}\right)\right] = 0$$

$$\sum_{s=0}^{\infty} \xi_p^s E_{it} \left[\Lambda_{t,t+s} Y_{t+s|t}\left(-\frac{X_{t,t+s}}{\lambda_{p,t+s}} + \left(\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}\right) \frac{MC_{i,t+s}}{P^*_{i,t}}\right)\right] = 0 \tag{B.5}$$

$$\sum_{s=0}^{\infty} \xi_p^s E_{it} \left[\Lambda_{t,t+s} Y_{t+s|t} \frac{X_{t,t+s}}{\lambda_{p,t+s}}\right] = \sum_{s=0}^{\infty} \xi_p^s E_{it} \left[\Lambda_{t,t+s} Y_{t+s|t}\left(\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}\right) \frac{MC_{i,t+s}}{P^*_{i,t}}\right] \tag{B.6}$$

B.4 Wage-setting problem

$$\min_{\{W^*_{h,t}\}} \quad -E_{ht}\left[\sum_{s=0}^{\infty} (\beta\xi_w)^s V\left(L_{h,t+s|t}\right)\right]$$

$$\text{s.t.} \quad L_{h,t+s|t} = \left(\frac{W^*_{h,t} X^w_{t,t+s}}{W_{t+s}}\right)^{-\frac{1+\mu_{w,t+s}}{\mu_{w,t+s}}} L_{t+s}$$

$$P_{t+s}C_{h,t+s} + P_{t+s}I_{h,t+s} + B_{h,t+s} + P_{t+s}a(U_{h,t+s})K_{h,t+s-1} + Q_{t+s+1,t}A_{h,t+s}$$
$$= R_{t+s-1}B_{h,t+s-1} + W_{h,t+s}L_{h,t+s|t} + R^k_{t+s}U_{h,t+s}K_{h,t+s-1} + P_{t+s}A_{h,t+s-1} + T_{h,t+s}$$

$$W_{h,t+s} = X^w_{t,t+s} W^*_{h,t}$$

$$X^w_{t,t+s} = \begin{cases} (\gamma\bar{\Pi}^{1-\iota_w})^s \prod_{j=1}^{s}\left(\Pi_{t+j-1}^{\iota_w}\right) & \text{if} \quad s \geq 1 \\ 1 & \text{if} \quad s = 0. \end{cases}$$

where $Y_{t+s|t}$ denote the output from period $t+s$ given that the last price update was done in period t.

The Lagrangean of this problem is given by

$$\mathcal{L} = \sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[V\left(L_{h,t+s|t}\right) + \Lambda_{h,t+s}\left(R_{t+s-1}B_{h,t+s-1} + X^w_{t,t+s}W^*_{h,t}L_{h,t+s|t} + R^k_{t+s}U_{h,t+s}K_{h,t-1} + \right.\right.$$
$$\left.\left. P_{t+s}A_{h,t+s-1} + T_{h,t+s} - P_{t+s}C_{h,t+s} - P_{t+s}I_{h,t+s} - B_{h,t+s} - P_{t+s}a(U_{h,t+s})K_{h,t+s-1} - Q_{t+s+1,t}A_{h,t+s}\right)\right]$$

Note that the Lagrangean multiplier of this problem is equal to the marginal utility of consumption, $\Lambda_{h,t+s}$.

The optimal condition for setting the wage of differentiated labor, $W^*_{h,t}$ is

$$0 = \sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[\left(\Lambda_{h,t+s}\left(X^w_{t,t+s}L_{h,t+s|t} - \frac{1+\mu^w_{t+s}}{\mu^w_{t+s}}L_{h,t+s|t}X^w_{t,t+s}\right) + \frac{1+\mu^w_{t+s}}{\mu^w_{t+s}}V'\left(L_{h,t+s}\right)\frac{L_{h,t+s|t}}{W^*_{h,t}}\right)\right]$$
$$0 = \sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[L_{h,t+s|t}\Lambda_{h,t+s}\left(\frac{X^w_{t,t+s}}{\mu^w_{t+s}} + \frac{1+\mu^w_{t+s}}{\mu^w_{t+s}}\frac{V'\left(L_{h,t+s}\right)}{\Lambda_{h,t+s}W^*_{h,t}}\right)\right]$$

Using the FOC from households' problem and the definition of the rate of marginal substitution between labor and consumption, one can see that

$$\frac{V'\left(L_{h,t+s|t}\right)}{\Lambda_{h,t+s}} = L^{\chi}_{h,t+s|t}(C_{h,t+s} - \varphi/\gamma C_{h,t+s-1})E_{h,t+s}[P_{t+s}]$$
$$= MRS_{h,t+s|t}E_{h,t+s}[P_{t+s}].$$

Thus, the wage-setting equation can be written as

$$\sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[L_{h,t+s|t}\Lambda_{h,t+s}\frac{X^w_{t,t+s}}{\mu^w_{t+s}}\right] = \sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[L_{h,t+s|t}\Lambda_{h,t+s}\frac{1+\mu^w_{t+s}}{\mu^w_{t+s}}\frac{MRS_{h,t+s|t}E_{h,t+s}[P_{t+s}]}{W^*_{h,t}}\right] \tag{B.7}$$

B.5 Labor packers

There is a continuum of households indexed by $h \in [0,1]$ who supplies differentiated labor services that are imperfect substitute for other households' labor services. As in **?**, it is assumed that there are "employment agencies" that combines households' labor supply using the following aggregator

$$L_t = \left(\int_0^1 L_{h,t}^{\frac{1}{1+\mu^w_t}}\,dh\right)^{1+\mu^w_t}, \tag{B.8}$$

where μ^w_t denote the agency's wage mark-up such that $log(1+\mu^w_t) = log(1+\mu^w_{t-1}) + \eta^w_t$. In turn, η^w_t follows the process

$$\eta^w_t = \rho_w\eta^w_{t-1} + \varepsilon^w_t, \quad \varepsilon^w_t \sim \mathcal{N}(0,\sigma^2_w). \tag{B.9}$$

Agencies pay for each household h their wage, $W_{h,t}$, and sell a homogeneous labor service to intermediate firms at a cost, W_t. Therefore, they maximize profits

$$W_t L_t - \int_0^1 W_{h,t} L_{h,t} dh \tag{B.10}$$

subject to (**??**). Thus, the demand labor for each household h is given by

$$L_{h,t} = \left(\frac{W_{h,t}}{W_t}\right)^{-\frac{1+\mu_t^w}{\mu_t^w}} L_t, \tag{B.11}$$

where $W_t = \left(\int_0^1 (W_{h,t})^{-\frac{1}{\mu_t^w}} dh\right)^{-\mu_t^w}$ is the nominal wage index. Given the optimal prices and the indexation rule, the price level has the following law of motion

$$W_t = \left[\xi_w \left(\gamma \Pi_{t-1}^{\iota_w} \bar{\Pi}^{1-\iota_w} W_{t-1}\right)^{-\frac{1}{\mu_t^w}} + (1-\xi_w)(W_t^*)^{-\frac{1}{\mu_t^w}}\right]^{-\mu_t^w}. \tag{B.12}$$

where $W_t^* = \int_0^1 W_{h,t}^* dh$

C Log-linearization of firms' problem

Denote $x_t = log(X_t) - log(\bar{X})$. We are using the Uhlig's method of linearization for most of equations.

C.1 Marginal cost

Equation (B.2)

$$\frac{E_{it}\left[W_t\right]}{E_{it}\left[R_t^k\right]} = \frac{1-\alpha}{\alpha}\frac{K_{i,t}}{L_{i,t}} \Rightarrow \frac{E_{it}\left[W_t/P_t\right]}{E_{it}\left[R_t^k/P_t\right]} = \frac{1-\alpha}{\alpha}\frac{K_{i,t}}{L_{i,t}} \Rightarrow \frac{E_{it}\left[\hat{W}_t\right]}{E_{it}\left[\hat{R}_t^k\right]} = \frac{1-\alpha}{\alpha}\frac{K_{i,t}}{L_{i,t}}$$

$$\frac{\bar{\hat{W}}}{\bar{\hat{R}}^k}\frac{E_{it}\left[e^{\hat{w}_t}\right]}{E_{it}\left[e^{\hat{r}_t^k}\right]} = \frac{1-\alpha}{\alpha}\frac{\bar{K}}{\bar{L}}e^{k_{i,t}-l_{i,t}}$$

$$\frac{E_{it}\left[e^{\hat{w}_t}\right]}{E_{it}\left[e^{\hat{r}_t^k}\right]} = e^{k_{i,t}-l_{i,t}}$$

where last equality uses the same equation in the steady-state.

Log-linearizing each term:

$$\frac{E_{it}\left[1+\hat{w}_t\right]}{E_{it}\left[1+\hat{r}_t^k\right]} = 1 + k_{i,t} - l_{i,t}$$
$$1 + E_{it}\left[\hat{w}_t\right] = (1 + k_{i,t} - l_{i,t})(1 + E_{it}\left[\hat{r}_t^k\right])$$
$$E_{it}\left[\hat{w}_t\right] = k_{i,t} - l_{i,t} + E_{it}\left[\hat{r}_t^k\right]$$

where last equality uses the fact that $x_t y_t \approxeq 0$ for any pair log-deviation from steady-state, (x_t, y_t).

Marginal cost (equation B.3)

$$MC_{i,t} = \left[\frac{(1-\alpha)W_t E_{it}\left[R_t^k\right] + \alpha R_t^k E_{it}\left[W_t\right]}{\alpha^\alpha(1-\alpha)^{1-\alpha}E_{it}\left[W_t\right]^\alpha E_{it}\left[R_t^k\right]^{1-\alpha}}\right]\frac{1}{A_t}$$
$$MC_{i,t}/P_t = \left[\frac{(1-\alpha)(W_t/P_t)E_{it}\left[R_t^k/P_t\right] + \alpha(R_t^k/P_t)E_{it}\left[W_t/P_t\right]}{\alpha^\alpha(1-\alpha)^{1-\alpha}E_{it}\left[W_t/P_t\right]^\alpha E_{it}\left[R_t^k/P_t\right]^{1-\alpha}}\right]\frac{1}{A_t}$$
$$\hat{MC}_{i,t} = \left[\frac{(1-\alpha)\hat{W}_t E_{it}\left[\hat{R}_t^k\right] + \alpha\hat{R}_t^k E_{it}\left[\hat{W}_t\right]}{\alpha^\alpha(1-\alpha)^{1-\alpha}E_{it}\left[\hat{W}_t\right]^\alpha E_{it}\left[\hat{R}_t^k\right]^{1-\alpha}}\right]\frac{1}{A_t}$$
$$\bar{MC}exp(mc_{i,t}) = \left[\frac{(1-\alpha)\bar{\hat{W}}\bar{\hat{R}}^k exp(\hat{w}_t + E_{it}\left[\hat{r}_t^k\right]) + \alpha\bar{\hat{W}}\bar{\hat{R}}^k exp(\hat{r}_t^k + E_{it}\left[\hat{w}_t\right])}{\alpha^\alpha(1-\alpha)^{1-\alpha}\bar{\hat{W}}^\alpha(\bar{\hat{R}}^k)^{1-\alpha}exp(\alpha E_{it}\left[\hat{w}_t\right] + (1-\alpha)E_{it}\left[\hat{r}_t^k\right])}\right]\frac{1}{exp(a_t)}$$
$$exp(mc_{it} + a_t) = \left[\frac{(1-\alpha)exp(\hat{w}_t + E_{it}\left[\hat{r}_t^k\right]) + \alpha exp(\hat{r}_t^k + E_{it}\left[\hat{w}_t\right])}{exp(\alpha E_{it}\left[\hat{w}_t\right] + (1-\alpha)E_{it}\left[\hat{r}_t^k\right])}\right]$$

where last equality uses that $\bar{MC} = \frac{\bar{\hat{W}}^{1-\alpha}(\bar{\hat{R}}^k)^\alpha}{\alpha^\alpha(1-\alpha)^{1-\alpha}}$

Log-linearizing each term:

$$1 + mc_{i,t} + a_t = \frac{(1-\alpha)(1 + \hat{w}_t + E_{it}\left[\hat{r}_t^k\right]) + \alpha(1 + \hat{r}_t^k + E_{it}\left[\hat{w}_t\right])}{1 + \alpha E_{it}\left[\hat{w}_t\right] + (1-\alpha)E_{it}\left[\hat{r}_t^k\right]}$$
$$(1 + mc_{i,t} + a_t)(1 + \alpha E_{it}\left[\hat{w}_t\right] + (1-\alpha)E_{it}\left[\hat{r}_t^k\right]) = (1-\alpha)(1 + \hat{w}_t + E_{it}\left[\hat{r}_t^k\right]) + \alpha(1 + \hat{r}_t^k + E_{it}\left[\hat{w}_t\right])$$
$$1 + mc_{i,t} + a_t + \alpha E_{it}\left[\hat{w}_t\right] + (1-\alpha)E_{it}\left[\hat{r}_t^k\right] = (1-\alpha)(1 + \hat{w}_t + E_{it}\left[\hat{r}_t^k\right]) + \alpha(1 + \hat{r}_t^k + E_{it}\left[\hat{w}_t\right])$$
$$mc_{i,t} = (1-\alpha)\hat{w}_t + \alpha\hat{r}_t^k - a_t$$

Note that $mc_{i,t}$ does not depend on i. It comes from the fact that the production function has constant returns to scale and, as it turns out, the heterogeneity of expectation does not play an role in the cost minimization problem up to the first order approximation. Thus, we can simply drop the i such that

$$mc_t = (1-\alpha)\hat{w}_t + \alpha \hat{r}^k_t - a_t$$

C.2 Production function of intermediate producers

Production function:

$$Y_{i,t} = e^{\eta^a_t} K^{\alpha}_{i,t} \left(\gamma^t L_{i,t}\right)^{1-\alpha} - \gamma^t \Phi_p$$

We first detrend the variables to get

$$\hat{Y}_{i,t} = e^{\eta^a_t} \hat{K}^{\alpha}_{i,t} L^{1-\alpha}_{i,t} - \Phi_p$$

We move the term Φ_p to the left-hand side of the equation and log-linearize it as follows

$$\log(\hat{Y}_{i,t} + \Phi_p) - \log(\bar{Y}_i + \Phi_p) = \eta^a + \alpha k_{i,t} + (1-\alpha) l_{i,t}$$

We may rearrange the left-hand side of the expression above to get

$$\log(\hat{Y}_{i,t} + \Phi_p) - \log(\bar{Y}_i + \Phi_p) = \frac{\bar{Y}_i}{\bar{Y}_i + \Phi_p} y_{i,t}$$

We conclude that

$$y_{i,t} = \frac{\bar{Y}_i + \Phi_p}{\bar{Y}_i} \left(\eta^a_t + \alpha k_{i,t} + (1-\alpha) l_{i,t}\right)$$

Integrating over i, we finaly get

$$y_t = \frac{\bar{Y} + \Phi_p}{\bar{Y}} \left(\eta^a_t + \alpha k_t + (1-\alpha) l_t\right) \tag{C.1}$$

C.3 Price index

Price index (equation B.4) requires a different log-linearization rule. Consider $X_t = Y_t^{Z_t}$. Then, $x_t = \bar{Z}(log(\bar{Y}) z_t + y_t)$ Applying this to equation (B.4):

$$\Pi_t^{-\frac{1}{\mu^p_t}} = \xi_p \left(\Pi^{\iota_p}_{t-1} \bar{\Pi}^{1-\iota_p}\right)^{-\frac{1}{\mu^p_t}} + (1-\xi_p)\left(\hat{P}^*_t\right)^{-\frac{1}{\mu^p_t}}$$

$$-\frac{1}{\bar{\lambda}_p}(-log(\bar{\Pi})\mu^p_t + \pi_t) = -\xi_p \left(\frac{1}{\bar{\lambda}_p}(-log(\bar{\Pi})\mu^p_t + \iota_p \pi_{t-1})\right) - (1-\xi_p)\frac{1}{\bar{\lambda}_p}(-log(\bar{\Pi})\mu^p_t + \hat{p}^*_t),$$

which rearranging leads to

$$\pi_t = \xi_p \iota_p \pi_{t-1} + (1-\xi_p)\hat{p}_t^* \tag{C.2}$$

C.4 Optimal price

Optimal price (equation B.6):

$$\begin{aligned}
\sum_{s=0}^{\infty} \xi_p^s E_{it}\left[\Lambda_{t,t+s} Y_{t+s|t} \frac{X_{t,t+s}}{\lambda_{p,t+s}}\right] &= \sum_{s=0}^{\infty} \xi_p^s E_{it}\left[\Lambda_{t,t+s} Y_{t+s|t} \left(\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}\right) \frac{MC_{i,t+s}}{P_{i,t}^*}\right] \\
\sum_{s=0}^{\infty} \xi_p^s E_{it}\left[\Lambda_{t,t+s} Y_{t+s|t} \frac{X_{t,t+s}}{\lambda_{p,t+s}}\right] &= \sum_{s=0}^{\infty} \xi_p^s E_{it}\left[\Lambda_{t,t+s} Y_{t+s|t} \left(\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}\right) \frac{MC_{i,t+s}}{P_{t+s}} \frac{P_{t-1}}{P_{i,t}^*} \frac{P_{t+s}}{P_{t-1}}\right] \\
\sum_{s=0}^{\infty} \xi_p^s E_{it}\left[\Lambda_{t,t+s} Y_{t+s|t} \frac{X_{t,t+s}}{\lambda_{p,t+s}}\right] &= \sum_{s=0}^{\infty} \xi_p^s E_{it}\left[\Lambda_{t,t+s} Y_{t+s|t} \left(\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}\right) \frac{\hat{MC}_{i,t+s}\Pi_{t-1,t+s}}{\hat{P}_{i,t}^*}\right]
\end{aligned}$$

where $\Pi_{t-1,t+s} = \frac{P_{t+s}}{P_{t-1}} = \sum_{j=0}^{s} \Pi_{t+j}$ and $\hat{P}_{i,t}^* = P_{i,t}^*/P_{t-1}$. If we stationarize $\hat{P}_{i,t}^*$ by P_t, we wouldn't be able to take it as known by firms, since P_t is unknown. Thus, we proceed dividing by P_{t-1}.

The equation above can be rewritten as

$$\begin{aligned}
&\sum_{s=0}^{\infty} \xi_p^s E_{it}\left[\Lambda_{t,t+s} Y_{t+s|t} \frac{X_{t,t+s}}{\lambda_{p,t+s}}\right] = \sum_{s=0}^{\infty} \xi_p^s E_{it}\left[\Lambda_{t,t+s} Y_{t+s|t} \left(\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}\right) \frac{\hat{MC}_{i,t+s}\Pi_{t-1,t+s}}{\hat{P}_{i,t}^*}\right] \\
&\sum_{s=0}^{\infty} \xi_p^s E_{it}\left[\Lambda_{t,t+s} Y_{t+s|t} \frac{X_{t,t+s}}{\lambda_{p,t+s}}\right] = -\sum_{s=0}^{\infty} \xi_p^s E_{it}\left[\Lambda_{t,t+s} Y_{t+s|t} \left(\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}\right) \frac{\hat{MC}_{i,t+s}\Pi_{t-1,t+s}}{\hat{P}_{i,t}^*}\right] \\
&\sum_{s=0}^{\infty} \xi_p^s E_{it}\left[\frac{\bar{\Lambda}_s \bar{Y} \bar{X}_s}{\bar{\lambda}_p} exp\left(\lambda_{t,t+s} + y_{t+s|t} + x_{t,t+s} - \tilde{\lambda}_{p,t+s}\right)\right] = \\
&\sum_{s=0}^{\infty} \xi_p^s E_{it}\left[\frac{\bar{\Lambda}_s \bar{Y}(1+\bar{\lambda}_p)\bar{MC}\bar{\Pi}^{s+1}}{\bar{\lambda}_p \bar{\Pi}} exp\left(\lambda_{t,t+s} + y_{t+s|t} + x_{t,t+s} + \tilde{\lambda}_{p,t+s} - \tilde{\lambda}_{p,t+s} + \hat{mc}_{i,t+s} + \pi_{t-1,t+s} - \hat{p}_{i,t}^*\right)\right]
\end{aligned}$$

where $\tilde{\lambda}_{p,t} = log(\mu_t^p) - log(\bar{\lambda}_p) \approx log(1+\mu_t^p) - log(1+\bar{\lambda}_p)$.

Note in the steady-state $\bar{\Lambda}_s = \left(\frac{\beta}{\bar{\Pi}}\right)^s$, $\bar{X}_s = \prod_{j=1}^{s}(\bar{\Pi}) = \bar{\Pi}^s$ and $\bar{MC} = 1/(1+\bar{\lambda}_p)$. Thus, the equation above can be written as

$$\begin{aligned}
&\sum_{s=0}^{\infty} (\beta\xi_p)^s E_{it}\left[exp\left(\lambda_{t,t+s} + y_{t+s|t} + x_{t,t+s} - \tilde{\lambda}_{p,t+s}\right)\right] = \\
&\sum_{s=0}^{\infty} (\beta\xi_p)^s E_{it}\left[exp\left(\lambda_{t,t+s} + y_{t+s|t} + \hat{mc}_{i,t+s} + \pi_{t-1,t+s} - \hat{p}_{i,t}^*\right)\right]
\end{aligned}$$

Loglinearizing leads to:

$$
\begin{aligned}
\sum_{s=0}^{\infty}(\beta\xi_p)^s E_{it}\left[1+\lambda_{t,t+s}+y_{t+s|t}+x_{t,t+s}-\tilde{\lambda}_{p,t+s}\right] &= \sum_{s=0}^{\infty}(\beta\xi_p)^s E_{it}[1+\lambda_{t,t+s}+y_{t+s|t}+\hat{m}c_{i,t+s} \\
&\quad +\pi_{t-1,t+s}-\hat{p}^*_{i,t}] \\
\sum_{s=0}^{\infty}(\beta\xi_p)^s E_{it}\left[x_{t,t+s}-\tilde{\lambda}_{p,t+s}\right] &= \sum_{s=0}^{\infty}(\beta\xi_p)^s E_{it}\left[\hat{m}c_{i,t+s}+\pi_{t-1,t+s}-\hat{p}^*_{i,t}\right] \\
\frac{\hat{p}^*_{i,t}}{1-\beta\xi_p} &= \sum_{s=0}^{\infty}(\beta\xi_p)^s E_{it}\left[\hat{m}c_{i,t+s}+\pi_{t-1,t+s}-x_{t,t+s}+\tilde{\lambda}_{p,t+s}\right] \\
\hat{p}^*_{i,t} &= (1-\beta\xi_p)\sum_{s=0}^{\infty}(\beta\xi_p)^s E_{it}\left[\hat{m}c_{i,t+s}+\pi_{t-1,t+s}-x_{t,t+s}+\tilde{\lambda}_{p,t+s}\right]
\end{aligned}
$$

Note that for $s \geq 1$ we have that

$$
\begin{aligned}
X_{t,t+s} &= \prod_{j=1}^{s}(\Pi_{t+j-1}^{\iota_p}\bar{\Pi}^{1-\iota_p}) \\
\bar{\Pi}^s exp(x_{t,t+s}) &= \bar{\Pi}^s exp\left(\iota_p\sum_{j=1}^{s}\pi_{t+j-1}\right)
\end{aligned}
$$

Log-linearizing we find that:

$$
x_{t,t+s} = \iota_p\sum_{j=1}^{s}\pi_{t+j-1} \tag{C.3}
$$

and $x_{t,t}=0$ by the definition of $X_{t,t+s}$. Analogously, we have that

$$
\pi_{t-1,t+s} = \sum_{j=0}^{s}\pi_{t+j}
$$

Using equations (C.3-C.4) and the fact that the marginal cost does not depend on i, the optimal price is given by

$$
\hat{p}^*_{i,t} = (1-\beta\xi_p)\sum_{s=0}^{\infty}(\beta\xi_p)^s E_{it}\left[\hat{m}c_{t+s}+\sum_{j=0}^{s}\pi_{t+j}-\iota_p\sum_{j=1}^{s}\pi_{t+j-1}+\tilde{\lambda}_{p,t+s}\right]
$$

Now we use the usual trick to rewrite $\hat{p}^*_{i,t}$ recursively.

$$
\begin{aligned}
\hat{p}^*_{i,t} &= (1-\beta\xi_p)\sum_{s=0}^{\infty}(\beta\xi_p)^s E_{it}\left[\hat{mc}_{t+s} + \sum_{j=0}^{s}\pi_{t+j} - \iota_p \sum_{j=1}^{s}\pi_{t+j-1} + \tilde{\lambda}_{p,t+s}\right] \\
&= (1-\beta\xi_p)E_{it}\left[\hat{mc}_t + \pi_t + \tilde{\lambda}_{p,t}\right] \\
&+ (1-\beta\xi_p)\sum_{s=1}^{\infty}(\beta\xi_p)^s E_{it}\left[\hat{mc}_{t+s} + \sum_{j=0}^{s}\pi_{t+j} - \iota_p \sum_{j=1}^{s}\pi_{t+j-1} + \tilde{\lambda}_{p,t+s}\right] \\
&= (1-\beta\xi_p)E_{it}\left[\hat{mc}_t + \pi_t + \tilde{\lambda}_{p,t}\right] \\
&+ (1-\beta\xi_p)\sum_{s=0}^{\infty}(\beta\xi_p)^{s+1} E_{it}\left[\hat{mc}_{i,t+s+1} + \sum_{j=0}^{s+1}\pi_{t+j} - \iota_p \sum_{j=1}^{s+1}\pi_{t+j-1} + \tilde{\lambda}_{p,t+s+1}\right] \\
&= (1-\beta\xi_p)E_{it}\left[\hat{mc}_t + \pi_t + \tilde{\lambda}_{p,t}\right] \\
&+ (1-\beta\xi_p)\beta\xi_p\sum_{s=0}^{\infty}(\beta\xi_p)^s E_{it}\left[\hat{mc}_{i,t+s+1} + \pi_t + \sum_{j=1}^{s+1}\pi_{t+j} - \iota_p\left(\pi_t + \sum_{j=2}^{s+1}\pi_{t+j-1}\right) + \tilde{\lambda}_{p,t+s+1}\right] \\
&= (1-\beta\xi_p)E_{it}\left[\hat{mc}_t + \pi_t + \tilde{\lambda}_{p,t}\right] + \beta\xi_p(1-\iota_p)E_{it}\left[\pi_t\right] \\
&+ (1-\beta\xi_p)\beta\xi_p\sum_{s=0}^{\infty}(\beta\xi_p)^s E_{it}\left[\hat{mc}_{i,t+s+1} + \sum_{j=0}^{s}\pi_{t+j+1} - \iota_p \sum_{j=1}^{s}\pi_{t+j} + \tilde{\lambda}_{p,t+s+1}\right]
\end{aligned}
$$

where the second equality separate the $s = 0$ term from the summation and the third redefines the summation counter as $s = s - 1$. The fourth equality separate the first term from the summations inside the brackets, and the fifth redefines the summation counter as $j = j - 1$ and solve the infinity sum for terms that do not depend on s.

Taking the expectation of the optimal price one-period ahead leads to

$$
\begin{aligned}
E_{it}[\hat{p}^*_{i,t+1}] &= (1-\beta\xi_p)\sum_{s=0}^{\infty}(\beta\xi_p)^s E_{it}\left[E_{i,t+1}\left[\hat{mc}_{i,t+s+1} + \sum_{j=0}^{s}\pi_{t+j+1} - \iota_p \sum_{j=1}^{s}\pi_{t+j} + \tilde{\lambda}_{p,t+s+1}\right]\right] \\
&= (1-\beta\xi_p)\sum_{s=0}^{\infty}(\beta\xi_p)^s E_{it}\left[\hat{mc}_{i,t+s+1} + \sum_{j=0}^{s}\pi_{t+j+1} - \iota_p \sum_{j=1}^{s}\pi_{t+j} + \tilde{\lambda}_{p,t+s+1}\right]
\end{aligned}
$$

where uses the law of iterated expectations (LIE) (which hold for i-th expectation but not for the average expectation). Thus, the optimal price can be written as

$$
\hat{p}^*_{i,t} = (1-\beta\xi_p)E_{it}\left[\hat{mc}_t + \tilde{\lambda}_{p,t}\right] + (1-\iota_p\beta\xi_p)E_{it}\left[\pi_t\right] + \beta\xi_p E_{it}\left[\hat{p}^*_{i,t+1}\right] \tag{C.4}
$$

Finally, we need to aggregate the optimal price to find the NKPC. Aggregating $\hat{p}^*_{i,t}$ leads to

$$\hat{p}^*_t = (1-\beta\xi_p)E^{(1)}_t\left[\hat{mc}_t + \tilde{\lambda}_{p,t}\right] + (1-\iota_p\beta\xi_p)E^{(1)}_t\left[\pi_t\right] + \beta\xi_p\int_0^1 E_{it}\left[\hat{p}^*_{i,t+1}\right]di$$

Denote the average of the individual expectations about their own future optimal price as $\mathbb{E}^{(1)}_t[\hat{p}^*_{i,t+1}] = \int_0^1 E_{it}\left[\hat{p}^*_{i,t+1}\right]di$. Note that it is different from the average expectation about the average optimal price $E^{(1)}_t\left[\hat{p}^*_{t+1}\right]$ (i.e., $\int_0^1 E_{it}\left[\hat{p}^*_{i,t+1}\right]di \neq E^{(1)}_t\left[\hat{p}^*_{t+1}\right]$).

Substituting into equation (C.2) leads to the NKPC under heterogeneous information and indexation

$$\begin{aligned}\pi_t &= \xi_p\iota_p\pi_{t-1} + (1-\xi_p)\left((1-\beta\xi_p)E^{(1)}_t\left[\hat{mc}_t + \tilde{\lambda}_{p,t}\right] + (1-\iota_p\beta\xi_p)E^{(1)}_t\left[\pi_t\right] + \beta\xi_p\mathbb{E}^{(1)}_t[\hat{p}^*_{i,t+1}]\right)\\ \pi_t &= \xi_p\iota_p\pi_{t-1} + \kappa_p\xi_pE^{(1)}_t\left[\hat{mc}_t + \tilde{\lambda}_{p,t}\right] + (1-\xi_p)(1-\iota_p\beta\xi_p)E^{(1)}_t\left[\pi_t\right] + (1-\xi_p)\beta\xi_p\mathbb{E}^{(1)}_t[\hat{p}^*_{i,t+1}]\end{aligned} \tag{C.5}$$

where $\kappa_p = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}$

C.5 Wage index

Wage index (24):

$$\begin{aligned}W_t &= \left[\xi_w\left(\gamma\Pi^{\iota_w}_{t-1}\bar{\Pi}^{1-\iota_w}W_{t-1}\right)^{-\frac{1}{\mu^w_t}} + (1-\xi_w)\left(W^*_t\right)^{-\frac{1}{\mu^w_t}}\right]^{-\mu^w_t}\\ \frac{W_t}{\gamma^tP_t} &= \left[\xi_w\left(\Pi^{\iota_w}_{t-1}\bar{\Pi}^{1-\iota_w}\frac{W_{t-1}}{\gamma^{t-1}P_{t-1}}\frac{P_{t-1}}{P_t}\right)^{-\frac{1}{\mu^w_t}} + (1-\xi_w)\left(\frac{W^*_t}{\gamma^tP_{t-1}}\frac{P_{t-1}}{P_t}\right)^{-\frac{1}{\mu^w_t}}\right]^{-\mu^w_t}\\ (\hat{W}_t\Pi_t)^{-\frac{1}{\mu^w_t}} &= \xi_w\left(\Pi^{\iota_w}_{t-1}\bar{\Pi}^{1-\iota_w}\hat{W}_{t-1}\right)^{-\frac{1}{\mu^w_t}} + (1-\xi_w)(\hat{W}^*_t)^{-\frac{1}{\mu^w_t}}\end{aligned}$$

Note that in the steady-state: $(\bar{\hat{W}}\bar{\Pi})^{-\frac{1}{\bar{\mu}^w}} = \xi_w\left(\bar{\Pi}\bar{\hat{W}}\right)^{-\frac{1}{\bar{\mu}^w}} + (1-\xi_w)\left(\bar{\hat{W}}^*\right)^{-\frac{1}{\bar{\mu}^w}}$, which implies that $\bar{\hat{W}}^* = \bar{\hat{W}}\bar{\Pi}$.

Following the same log-linearization technique from the price index leads to

$$\begin{aligned}-\frac{1}{\bar{\mu}^w}(-log(\bar{\Pi}\bar{\hat{W}})\mu^w_t + \hat{w}_t + \pi_t) = -\xi_w\left(\frac{1}{\bar{\mu}^w}(-log(\bar{\Pi}\bar{\hat{W}})\mu^w_t + \iota_w\pi_{t-1} + \hat{w}_{t-1})\right)\\ -(1-\xi_w)\frac{1}{\bar{\mu}^w}(-log(\bar{\hat{W}}^*)\mu^w_t + \hat{w}^*_t),\end{aligned}$$

which rearranging leads to

$$\hat{w}_t + \pi_t = \xi_w(\iota_w\pi_{t-1} + \hat{w}_{t-1}) + (1-\xi_w)\hat{w}^*_t \tag{C.6}$$

where $\hat{w}_t^* = \int_0^1 \hat{w}_{h,t}^* dh$.

C.6 Optimal wage

Now we turn to the optimal wage (39)

$$\sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[L_{h,t+s|t}\Lambda_{h,t+s}\frac{X_{t,t+s}^w}{\mu_{t+s}^w}\right] = \sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[L_{h,t+s|t}\Lambda_{h,t+s}\frac{1+\mu_{t+s}^w}{\mu_{t+s}^w}\frac{MRS_{h,t+s|t}E_{h,t+s}[P_{t+s}]}{W_{h,t}^*}\right]$$
$$\sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[L_{h,t+s|t}\frac{\Lambda_{h,t+s}\gamma^{t+s}P_{t-1}}{\gamma^{t+s}P_{t+s}}\frac{X_{t,t+s}^w}{\mu_{t+s}^w}\right]$$
$$= \sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[L_{h,t+s|t}\frac{\Lambda_{h,t+s}\gamma^{t+s}P_{t-1}}{\gamma^{t+s}P_{t+s}}\frac{1+\mu_{t+s}^w}{\mu_{t+s}^w}\frac{MRS_{h,t+s|t}P_{t-1}\gamma^t E_{h,t+s}[P_{t+s}]}{\gamma^t W_{h,t}^* P_{t-1}}\right]$$
$$\sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[L_{h,t+s|t}\frac{\hat{\Lambda}_{h,t+s}}{\gamma^s}\frac{X_{t,t+s}^w}{\mu_{t+s}^w\Pi_{t-1,t+s}}\right]$$
$$= \sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[L_{h,t+s|t}\hat{\Lambda}_{h,t+s}\frac{1+\mu_{t+s}^w}{\mu_{t+s}^w}\frac{M\hat{R}S_{h,t+s|t}E_{h,t+s}[\Pi_{t-1,t+s}]}{\hat{W}_{h,t}^*\Pi_{t-1,t+s}}\right]$$

This equation in the steady-state implies that

$$\sum_{s=0}^{\infty}(\beta\xi_w)^s\bar{L}\frac{\bar{\hat{\Lambda}}}{\gamma^s}\frac{(\gamma\bar{\Pi})^s}{\bar{\mu}^w\bar{\Pi}^s} = \sum_{s=0}^{\infty}(\beta\xi_w)^s\bar{L}\bar{\hat{\Lambda}}\frac{1+\bar{\mu}^w}{\bar{\mu}^w}\frac{M\bar{\hat{R}}S\bar{\Pi}^{s+1}}{\bar{\hat{W}}\Pi^{s+1}}$$
$$\sum_{s=0}^{\infty}(\beta\xi_w)^s = \sum_{s=0}^{\infty}(\beta\xi_w)^s\frac{(1+\bar{\mu}^w)M\bar{\hat{R}}S}{\bar{\hat{W}}}$$
$$\bar{\hat{W}} = (1+\bar{\mu}^w)M\bar{\hat{R}}S$$

Now we turn to the log-linearization:

$$\sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[exp\left(l_{h,t+s|t} + \hat{\lambda}_{h,t+s} + x^w_{t,t+s} - \pi_{t-1,t+s} - \tilde{\mu}^w_{t+s}\right)\right]$$
$$= \sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[exp\left(l_{h,t+s|t} + \hat{\lambda}_{h,t+s} + m\hat{r}s_{h,t+s|t} + E_{h,t+s}[\pi_{t-1,t+s}] - \hat{w}^*_{h,t} - \pi_{t-1,t+s}\right)\right]$$
$$\sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[1 + l_{h,t+s|t} + \hat{\lambda}_{h,t+s} + x^w_{t,t+s} - \pi_{t-1,t+s} - \tilde{\mu}^w_{t+s}\right]$$
$$= \sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[1 + l_{h,t+s|t} + \hat{\lambda}_{h,t+s} + m\hat{r}s_{h,t+s|t} - \hat{w}^*_{h,t}\right]$$
$$\hat{w}^*_{h,t} = (1-\beta\xi_w)\sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[m\hat{r}s_{h,t+s|t} + \pi_{t-1,t+s} - x^w_{t,t+s} + \tilde{\mu}^w_{t+s}\right]$$
$$\hat{w}^*_{h,t} = (1-\beta\xi_w)\sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[m\hat{r}s_{h,t+s|t} + \sum_{j=0}^{s}\pi_{t+j} - \iota_w\sum_{j=1}^{s}\pi_{t+j-1} + \tilde{\mu}^w_{t+s}\right]$$

where $\tilde{\mu}^w_t = log(\mu^w_t) - log(\bar{\mu}^w) \approx log(1+\mu^w_t) - log(1+\bar{\mu}^w)$.

Using the labor demand (14) at period $t+s$ for a given wage set at t, we have that

$$L_{h,t+s|t} = \left(\frac{W^*_{h,t}X^w_{t,t+s}}{W_{t+s}}\right)^{-\frac{1+\mu^w_{t+s}}{\mu^w_{t+s}}} L_{t+s}$$
$$L_{h,t+s|t} = \left(\frac{\hat{W}^*_{h,t}}{\gamma^{t+s}P_{t-1}}\frac{\gamma^{t+s}P_{t+s}}{\hat{W}_{t+s}}\frac{P_{t-1}}{P_{t+s}}X^w_{t,t+s}\right)^{-\frac{1+\mu^w_{t+s}}{\mu^w_{t+s}}} L_{t+s}$$
$$L_{h,t+s|t} = \left(\frac{\hat{W}^*_{h,t}X^w_{t,t+s}}{\gamma^s\hat{W}_{t+s}\Pi_{t-1,t+s}}\right)^{-\frac{1+\mu^w_{t+s}}{\mu^w_{t+s}}} L_{t+s}$$

Since $\bar{X}^w_s = (\gamma\bar{\Pi})^s$, $\bar{\Pi}_{-1,s} = \bar{\Pi}^{s+1}$, and $\bar{\hat{W}}^* = \bar{\hat{W}}\bar{\Pi}$, we have that $\bar{L}_h = \bar{L}$. Log-linearizing implies

$$l_{h,t+s|t} = -\frac{1+\bar{\mu}^w}{\bar{\mu}^w}\left[\hat{w}^*_{h,t} + \iota_w\sum_{j=1}^{s}\pi_{t+j-1} - \hat{w}_{t+s} - \sum_{j=0}^{s}\pi_{t+j}\right] + l_{t+s} \tag{C.7}$$

where the first equality uses that $log\left(\frac{\bar{\hat{W}}^*\bar{X}^w_s}{\bar{\hat{W}}\gamma^s\bar{\Pi}^{s+1}}\right) = log(1) = 0$. and the second uses that – analogously to the indexation scheme for prices – $x^w_{t,t+s} = \iota_w\sum_{j=1}^{s}\pi_{t+j-1}$.

The MRS at period $t+s$ for a given wage set at t is given by

$$\begin{aligned}
m\hat{r}s_{h,t+s|t} &= \left(\frac{1}{1-\varphi/\gamma}\right)(c_{h,t+s} - \varphi/\gamma c_{h,t+s-1}) + \chi l_{t+s|t} \\
&= \left(\frac{1}{1-\varphi/\gamma}\right)(c_{h,t+s} - \varphi/\gamma c_{h,t+s-1}) \\
&+ \chi\left(-\frac{1+\bar{\mu}^w}{\bar{\mu}^w}\left[\hat{w}^*_{h,t} + \iota_w \sum_{j=1}^{s} \pi_{t+j-1} - \hat{w}_{t+s} - \sum_{j=0}^{s} \pi_{t+j}\right] + l_{t+s}\right)
\end{aligned}$$

Using the fact that $c_{h,t} = c_t$ for all h and the average MSR, mrs_t, we have that

$$m\hat{r}s_{h,t|t+s} = m\hat{r}s_{t+s} - \chi\theta_w\left[\hat{w}^*_{h,t} + \iota_w \sum_{j=1}^{s} \pi_{t+j-1} - \hat{w}_{t+s} - \sum_{j=0}^{s} \pi_{t+j}\right]$$

where $\theta_w \equiv \frac{1+\bar{\mu}^w}{\bar{\mu}^w}$.

Substituting it back into the optimal wage

$$\hat{w}^*_{h,t} = \frac{(1-\beta\xi_w)}{1+\chi\theta_w}\sum_{s=0}^{\infty}(\beta\xi_w)^s E_{ht}\left[m\hat{r}s_{t+s} + \chi\theta_w\hat{w}_{t+s} + (1+\chi\theta_w)\left(\sum_{j=0}^{s}\pi_{t+j} - \iota_w\sum_{j=1}^{s}\pi_{t+j-1}\right) + \tilde{\mu}^w_{t+s}\right]$$

Following similar steps of the optimal price equation, we can rewrite the optimal wage as

$$\begin{aligned}
\hat{w}^*_{h,t} &= \frac{(1-\beta\xi_w)}{1+\chi\theta_w}E_{ht}\left[(m\hat{r}s_t + w_t + (1+\chi\theta_w)\pi_t + \tilde{\mu}^w_t)\right] + (1-\iota_w)\beta\xi_w E_{ht}\left[\pi_t\right] + \beta\xi_w E_{ht}\left[\hat{w}^*_{h,t+1}\right] \\
&= \frac{(1-\beta\xi_w)}{1+\chi\theta_w}E_{ht}\left[(m\hat{r}s_t + w_t + \tilde{\mu}^w_t)\right] + (1-\iota_w\beta\xi_w)E_{ht}\left[\pi_t\right] + \beta\xi_w E_{ht}\left[\hat{w}^*_{h,t+1}\right]
\end{aligned}$$

Integrating over h:

$$\hat{w}^*_t = \frac{(1-\beta\xi_w)}{1+\chi\theta_w}E^{(1)}_t\left[(m\hat{r}s_t + w_t + \tilde{\mu}^w_t)\right] + (1-\iota_w\beta\xi_w)E^{(1)}_t\left[\pi_t\right] + \beta\xi_p\int_0^1 E_{ht}\left[\hat{w}^*_{h,t+1}\right]dh \qquad \text{(C.8)}$$