

Expectations and Frictions: Lessons from a Quantitative Dispersed Information Model*

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Abstract

We examine the macroeconomic implications of information frictions in a medium-scale DSGE model. We develop a solution method for dispersed-information DSGE models that accommodates endogenous states, and estimate the model using Bayesian methods using macroeconomic and expectations data. We assess the relative importance of standard rigidities, learning, and higher-order expectations. Fitting expectations data requires strong information frictions, which dampen general equilibrium effects and shift the relative importance of shocks driving fluctuations. Information frictions are crucial for hump-shaped inflation responses and can replace most standard frictions. Habit formation remains a key exception, becoming more important when expectations data are included.

Keywords: Business cycles, DSGE models, information frictions, higher-order beliefs.

JEL Codes: E31, E32, D83, D84.

1 Introduction

Workhorse medium-scale models in the tradition of [Christiano et al. \(2005\)](#) and [Smets and Wouters \(2007\)](#) reproduce the persistent and hump-shaped responses of macroeconomic aggregates by relying on habit formation, adjustment costs, and indexation, while retaining

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full-information rational expectations. These models are usually estimated on macroeconomic variables alone, not confronting that assumption with data on expectations, even though expectations are a key channel through which shocks propagate.¹ Evidence from survey forecasts, however, shows systematic departures from full-information rational expectations, in the direction that slows the transmission of shocks: average forecasts underreact to news (Coibion and Gorodnichenko, 2012, 2015).²

These empirical tests were motivated by a theoretical literature showing that information frictions generate sluggish responses to shocks, serving as an alternative source of persistence (Mankiw and Reis, 2002; Woodford, 2002; Sims, 2003). Angeletos and Huo (2021) sharpen this into an equivalence between incomplete information and standard inertial mechanisms in a simple framework. To our knowledge, the relative importance of these two types of frictions has not been quantified using both macroeconomic and expectations data.

This paper measures the extent to which information frictions can substitute for these inertial mechanisms. We estimate a medium-scale DSGE model with dispersed information alongside habit formation, adjustment costs, and indexation, using data on U.S. macroeconomic aggregates and survey forecast revisions. Solving this model requires accounting for endogenous states as well as the hierarchy of beliefs arising from agents' expectations of others' decisions (Townsend, 1983). We contribute on two fronts: an extension of existing truncation methods that accommodates endogenous states and a quantitative assessment of the role of information frictions in business cycle dynamics.

Fitting the expectations data requires strong information frictions: agents incorporate only a quarter to a half of most shocks into their beliefs on impact. Macroeconomic aggregates alone identify these frictions weakly. These frictions dampen general equilibrium feedback, shift the relative importance of shocks driving fluctuations, and generate hump-shaped inflation responses that the estimated full-information model does not produce. Information frictions can substitute for most standard inertial mechanisms. Habit formation is the exception: removing it substantially lowers the marginal likelihood when expectations data are included. Without habits, the model relies on a more persistent preference shock to match consumption persistence, at the cost of excessively persistent revisions to consumption expectations.

¹Del Negro and Schorfheide (2013); Smets et al. (2014); Müller et al. (2022) include survey expectations among the observables when estimating DSGE models under full information. See Milani (2023) for a review of the use of expectational data in DSGE models.

²The underreaction of consensus forecasts coexists with overreaction at the individual level (Bordalo et al., 2020) and with over-extrapolation to recent events (Kohlhas and Walther, 2021).

Solution method. Our methodological contribution is a solution algorithm for log-linear DSGE models with dispersed and exogenous information that remains feasible for larger-scale applications. The method bridges Uhlig’s (2001) undetermined coefficient method for full-information models with the solution methods from Nimark (2008) and Melosi (2017) for dispersed information models that truncate the hierarchy of beliefs to a finite order.

Our method advances these approaches in two ways. First, it accommodates endogenous state variables in the log-linearized equilibrium conditions under dispersed information, expanding the class of models that can be solved via the truncation approach. Second, it employs a representation of the system of equations that depends only on average expectations rather than requiring equilibrium conditions written in terms of higher-order expectations. These two features simplify the equilibrium conditions, so that the implementation is only marginally more involved than solving a standard full-information model. The algorithm is sufficiently fast to perform Bayesian estimation with standard techniques.

Application in a quantitative model. Our applied contribution is to estimate a canonical medium-scale DSGE model along the lines of Smets and Wouters (2007) (henceforth, SW) augmented with dispersed information. In the model, neither firms nor households observe the underlying aggregate shocks perfectly. Each agent receives noisy idiosyncratic signals about the shocks and forms expectations rationally conditional on them. Unlike prior work that estimates small-scale New Keynesian models under dispersed information (Melosi, 2014, 2017), our framework nests the standard frictions of the SW model alongside information frictions. We estimate it on U.S. macroeconomic time series and average forecast revisions from the Survey of Professional Forecasters (SPF).

We use this framework to examine how shocks propagate when agents are imperfectly informed, which shocks drive real and nominal fluctuations, the relative roles of learning about shocks and higher-order expectations, and whether information frictions can substitute for standard inertial mechanisms in fitting both macroeconomic and expectations data. To answer these questions, we compare two specifications, a baseline model with dispersed information (DI) and a restricted version under full information (FI), each estimated on two datasets: U.S. macroeconomic aggregates only, and the same aggregates augmented with survey expectations data. This design separates the effect of including expectations data in the estimation from the effect of including information frictions in the model.

Estimated degree of information frictions. We quantify information frictions about each structural shock using the steady-state Kalman gain, the weight agents place on new information when updating their beliefs. Our simple information structure implies that the

Kalman gain ranges from one (full information) to zero (“no information”).

By incorporating expectations data into the estimation, we document three key findings. First, the DI model fits the expectations data substantially better than the FI model, with a marginal likelihood over 200 log points higher. Second, we find strong and pervasive information frictions. Most shocks have precisely estimated Kalman gains that lie roughly between 0.25 and 0.50, implying that agents’ beliefs about a given shock incorporate only a quarter to a half of its realization, with the remaining weight on their previous beliefs. Two exceptions sit at opposite ends of the scale. Learning about monetary policy shocks is faster, with an estimated gain of 0.72 whose 90% credible interval excludes the full-information benchmark of one. At the other extreme, the gain on the TFP shock is estimated at 0.01, implying that beliefs about technology barely respond to new information.³

We validate these structural estimates using a regression based on the joint behavior of observed average forecast errors and revisions, along the lines of [Coibion and Gorodnichenko \(2015\)](#). We compute the implied information frictions from data simulated at the posterior mode of the DI model and compare them with those obtained from the same regression estimated on SPF data. The model-implied Kalman gains are close to the SPF estimates, whereas simulations based on the prior, or on estimates that use macroeconomic data only, perform markedly worse.

Third, expectations data are essential for identifying information frictions. Estimated on macroeconomic aggregates alone, the model delivers a different picture: the Kalman gains of most shocks rise sharply, with some close to the full-information benchmark, while the monetary policy shock, which has the highest Kalman gain with expectations data, has the lowest without them.

Can information frictions substitute for standard frictions? Our framework provides a natural test of the conjecture that information frictions can replace standard inertial mechanisms, as it nests both types of frictions and estimates them jointly under loose priors. We re-estimate the model under specifications that remove selected frictions, comparing marginal likelihoods to the baseline.

Removing all information frictions (the full-information benchmark) reduces the log marginal likelihood by about 219 log points, more than two and a half times the 83 log point loss from removing all standard frictions. Most standard inertial mechanisms become non-essential once information frictions are introduced: shutting down price indexation or

³The technology shock is also estimated to be nearly permanent. Combined with the low gain, this implies that beliefs are almost entirely anchored on their own past. This configuration has a limited quantitative footprint: technology shocks account for a small share of the forecast error variance of every observable except hours worked at longer horizons.

replacing investment adjustment costs with capital adjustment costs each slightly improves fit, while shutting down wage indexation worsens it only marginally.

Habit formation is the main exception, reflecting a central empirical tension: macroeconomic aggregates are highly persistent, whereas expectation revisions underreact to shocks and are only weakly persistent. Strong information frictions explain this underreaction, but they also make it difficult to match aggregate sluggishness without generating overly persistent revisions. Whether additional inertial mechanisms are needed then depends on shock persistence. Capital adjustment costs, combined with a more persistent investment shock and somewhat weaker information frictions that raise the persistence of expectations only modestly, balance hump-shaped investment dynamics with weakly persistent revisions. For consumption, no such alternative exists: without habits, matching consumption persistence requires a highly persistent preference shock, which generates overly persistent revisions that the expectations data reject.

Business cycle dynamics under dispersed information. Having established the relative importance of each type of friction, we turn to how incomplete information shapes business cycle dynamics. For both the DI and FI models, the variance decomposition identifies investment-specific technological and government expenditure shocks as the primary drivers of output, and price mark-up shocks as the dominant source of inflation variation. Information frictions instead concentrate each variable’s decomposition on its own shock, the one that enters its equilibrium condition directly. Agents respond less to aggregate conditions they do not fully observe, which dampens general equilibrium feedback and strengthens partial equilibrium effects. For instance, relative to full information, price mark-up shocks account for a larger share of inflation variation, and investment shocks for a larger share of investment variation.

We compare impulse responses from the DI and FI models to investment-specific technological and cost-push (price mark-up) shocks, the main drivers of output and inflation fluctuations, respectively. Information frictions produce a hump-shaped inflation response to the cost-push shock that the FI model cannot replicate, because the shock is persistent and price indexation is negligible. The investment shock is less persistent, and aggregate responses to it are qualitatively similar across models. For both shocks, DI revisions are attenuated on impact and unwind gradually, whereas FI revisions jump and return to zero immediately.

We decompose these effects into first-order uncertainty (gradual learning about shocks) and higher-order uncertainty (uncertainty about others’ beliefs). For both shocks, first-order learning accounts for essentially all of the attenuation and delay in macroeconomic variables.

Higher-order beliefs matter little for aggregates for two reasons. First, most forward-looking terms are agents' expectations of their own future choices, which involve no strategic uncertainty about others' beliefs. Second, strategic uncertainty enters mostly through the dependence of inflation and the real wage on the average expectation of the same variable, and the coefficients on these terms are small because the estimated price and wage Phillips curves are flat. Higher-order beliefs matter instead for expectations dynamics: they account for about a third of the underreaction of forecast revisions on impact and delay their peak, which is key to the DI model's advantage over the FI model in fitting the expectations data.

Related literature. Our paper relates to a literature that develops solution methods for DSGE models with dispersed information. We extend the truncation of the belief hierarchy in [Nimark \(2008\)](#) and [Melosi \(2017\)](#). An alternative uses frequency domain techniques: [Huo and Takayama \(2025\)](#) combine the Wiener-Hopf prediction formula with the Kalman filter to obtain an exact finite-state representation of the equilibrium, which they apply to a small-scale beauty contest model and a HANK model with incomplete information.⁴ Truncation instead maps the model into the state-space form used to estimate full-information DSGE models and keeps likelihood evaluation fast enough for Bayesian estimation at medium scale.

Dispersed information is one of several departures from full-information rational expectations that can generate the persistence usually attributed to standard frictions.⁵ This substitution has also been explored using models with imperfect common information ([Collard et al., 2009](#); [Collard and Dellas, 2010](#); [Levine et al., 2012](#)) and adaptive learning ([Milani, 2007, 2011](#); [Slobodyan and Wouters, 2012](#)).⁶ Relatedly, other departures from full-information rational expectations have been added to medium-scale DSGE models and disciplined by expectations data: imperfect information ([Del Negro and Eusepi, 2011](#)), diagnostic expectations ([L'Huillier et al., 2024](#); [Bianchi et al., 2024](#)), and adaptive learning ([Ormeño and Molnár, 2015](#); [Milani, 2017](#); [Rychalovska et al., 2026](#)).

Outline. The paper proceeds as follows. Section 2 presents the model. Section 3 describes the solution method. Section 4 describes the data and the estimation. Section 5 examines

⁴For other papers using frequency domain techniques to solve dispersed information models, see [Kasa \(2000\)](#); [Kasa et al. \(2014\)](#); [Rondina and Walker \(2021\)](#) and references therein. Alternatively, [Auclert et al. \(2020\)](#) use a sequence-space approach to estimate a HANK model under sticky information.

⁵A complementary literature studies business cycles driven by belief fluctuations and higher-order uncertainty, through sentiment and confidence shocks ([Angeletos and La'O, 2013](#); [Angeletos et al., 2018](#)), endogenous signal informativeness ([Nimark, 2014](#)), and general information structures ([Chahrour and Ulbricht, 2023](#)). We restrict fluctuations to fundamental shocks, so that the comparison across frictions is not confounded by belief shocks that could themselves account for part of the dynamics.

⁶[Milani \(2011\)](#) uses survey forecast data in the estimation while [Slobodyan and Wouters \(2012\)](#) use it only for validation.

the quantitative implications of information frictions. Section 6 concludes.

2 Model

The model is a standard medium-scale DSGE model along the lines of SW. The innovation of our application is the introduction of dispersed information to this environment. Specifically, households and firms do not observe perfectly the shocks hitting the economy. Instead, they receive noisy idiosyncratic signals about them and form expectations rationally conditional on this information.

Timing

Time is discrete and each period has two stages. In stage 1, shocks and signals realize, intermediate goods firms set optimal prices, and households choose consumption, investment, installed capital, and its utilization level, and set optimal wages, based on information from their signals. In stage 2, rental rates and wages of differentiated labor are revealed and production takes place for final goods, intermediate goods and labor packers.

This timing protocol is standard in the literature and ensures two features: intermediate firms and households use only information from their signals to form expectations and all markets clear.⁷ Competitive final good firms ensure that the supply of final good matches consumption and investment demands. Given prices of intermediate goods, differentiated wages, and rental rate chosen at stage 1, firms allocate capital and labor to accommodate the demand from final good firms. Finally, given the wages set at the first stage, labor packers aggregate the differentiated labor services from households and supply homogeneous labor to intermediate firms such that the labor market clears.

Final good firms

The homogeneous final good Y_t is a bundle of intermediate goods, $Y_{i,t}$, where the index $i \in [0, 1]$ denotes the continuum of intermediate firms. The production function is given by

$$Y_t = \left(\int_0^1 (Y_{i,t})^{\frac{1}{1+\mu_t^p}} di \right)^{1+\mu_t^p}, \quad (1)$$

where μ_t^p is the time-varying price mark-up of intermediate goods following the process $\log(1 + \mu_t^p) = \log(1 + \bar{\mu}^p) + x_t^p$, and $\bar{\mu}^p$ is the steady-state value of the price mark-up. The

⁷See for instance [Nimark \(2008\)](#) and [Angeletos and La'O \(2011\)](#).

price mark-up shock follows $x_t^p = \rho_p x_{t-1}^p + \varepsilon_t^p$, where $\varepsilon_t^p \sim \mathcal{N}(0, \sigma_p^2)$.

Final goods firms operate in a perfectly competitive market and sell the final goods to households for a price P_t . They solve a standard profit maximization problem

$$P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di \quad (2)$$

subject to (1). From the first-order conditions and the zero-profit condition, we derive the demand for intermediate production

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\frac{1+\mu_t^p}{\mu_t^p}} Y_t, \quad (3)$$

where $P_t = \left(\int_0^1 (P_{i,t})^{-\frac{1}{\mu_t^p}} di \right)^{-\mu_t^p}$ is the price level.

Intermediate good firms

Each intermediate producer is a monopolistic competitive firm $i \in [0, 1]$ that uses labor and capital to produce its goods. Good i is produced using technology

$$Y_{i,t} = e^{x_t^a} (K_{i,t})^\alpha (\gamma^t L_{i,t})^{1-\alpha} - \gamma^t \Phi_p, \quad (4)$$

where γ represents the labor-augmenting deterministic growth rate of productivity and x_t^a is a total factor productivity shock. $K_{i,t}$ and $L_{i,t}$ denote the amount of capital and labor demanded by firm i at period t respectively, and Φ_p is a fixed cost included in the production function.⁸ The productivity shock follows $x_t^a = \rho_a x_{t-1}^a + \varepsilon_t^a$, where $\varepsilon_t^a \sim \mathcal{N}(0, \sigma_a^2)$.

Intermediate firms are subject to a [Calvo \(1983\)](#)-like pricing friction: in every period, only a fraction $1 - \xi_p$ of firms can adjust prices. Firms not allowed to reset prices apply the following indexation rule

$$P_{i,t} = (\Pi_{t-1})^{\iota_p} (\bar{\Pi})^{1-\iota_p} P_{i,t-1}, \quad (5)$$

where $\Pi_t \equiv P_t/P_{t-1}$ is gross inflation and $\bar{\Pi}$ is its steady-state value. Firms that are able to reoptimize prices choose the level $P_{i,t}^*$ that maximizes their expected discounted flow of profits given by

$$E_{i,t} \sum_{s=0}^{\infty} (\beta \xi_p)^s \Lambda_{t,t+s} \left[(X_{t,t+s} P_{i,t}^* - MC_{i,t+s}) Y_{i,t+s} \right], \quad (6)$$

subject to $Y_{i,t+s} = \left(\frac{X_{t,t+s} P_{i,t}^*}{P_{t+s}} \right)^{-\frac{1+\mu_{t+s}^p}{\mu_{t+s}^p}} Y_{t+s}$. $MC_{i,t}$ is the marginal cost of firm i at period t .

⁸Scaling the fixed cost by γ^t prevents it from decreasing over time in terms of output.

$\Lambda_{t,t+s}$ is the household's stochastic discount factor between periods t and $t+s$ and $X_{t,t+s}$ is the indexation between the same periods given by

$$X_{t,t+s} = \begin{cases} \bar{\Pi}^{(1-\iota_p)s} \Pi_{j=1}^s \left(\Pi_{t+j-1}^{\iota_p} \right) & \text{if } s \geq 1 \\ 1 & \text{if } s = 0. \end{cases} \quad (7)$$

Given the optimal prices and the indexation rule, the price level follows

$$P_t = \left[\xi_p \left(\Pi_{t-1}^{\iota_p} \bar{\Pi}^{1-\iota_p} P_{t-1} \right)^{-\frac{1}{\mu_t^p}} + (1 - \xi_p) \int_0^1 \left(P_{i,t}^* \right)^{-\frac{1}{\mu_t^p}} di \right]^{-\mu_t^p}. \quad (8)$$

Labor packers

There is a continuum of households indexed by $h \in [0, 1]$, each supplying differentiated labor services. Following [Erceg et al. \(2000\)](#), there are labor packers that hire labor from the households and aggregate them according to

$$L_t = \left(\int_0^1 L_{h,t}^{\frac{1}{1+\mu_t^w}} dh \right)^{1+\mu_t^w}, \quad (9)$$

where μ_t^w denotes the agency's wage mark-up such that $\log(1 + \mu_t^w) = \log(1 + \bar{\mu}^w) + x_t^w$, and $\bar{\mu}^w$ is the steady-state value of the wage mark-up. The shock follows the process $x_t^w = \rho_w x_{t-1}^w + \varepsilon_t^w$, where $\varepsilon_t^w \sim \mathcal{N}(0, \sigma_w^2)$.

Labor packers pay the wage $W_{h,t}$ for each household h , and sell a homogeneous labor service to intermediate firms at a cost W_t . Labor packers maximize profits

$$W_t L_t - \int_0^1 W_{h,t} L_{h,t} dh \quad (10)$$

subject to (9). Thus, the labor demand for each household h 's labor service is given by

$$L_{h,t} = \left(\frac{W_{h,t}}{W_t} \right)^{-\frac{1+\mu_t^w}{\mu_t^w}} L_t, \quad (11)$$

where $W_t = \left(\int_0^1 (W_{h,t})^{-\frac{1}{\mu_t^w}} dh \right)^{-\mu_t^w}$ is the nominal wage index.

Households

Household $h \in [0, 1]$ maximizes its expected utility choosing consumption $(C_{h,t})$, holdings of government bonds $(B_{h,t})$, installed capital level $(K_{h,t})$ and its utilization rate $(U_{h,t})$. The

capital rented to firms, K_t^u , is determined by the installed capital and the utilization rate. The objective function that each household h optimizes is given by

$$U = E_{h,t} \left[\sum_{s=0}^{\infty} \beta^s e^{x_{t+s}^c} \left(\ln (C_{h,t+s} - \varphi C_{h,t+s-1}) - \frac{L_{h,t+s}^{1+\chi}}{1+\chi} \right) \right], \quad (12)$$

where $C_{h,t}$ is consumption and $L_{h,t}$ denotes the supply of differentiated labor services of household h at period t . Households' preferences display internal habit persistence, captured by the parameter φ , while χ is the inverse of the Frisch elasticity of labor supply. $E_{h,t}[\cdot]$ is the expectation operator conditional on household h 's information set, and the preference shock follows $x_t^c = \rho_c x_{t-1}^c + \varepsilon_t^c$, where $\varepsilon_t^c \sim \mathcal{N}(0, \sigma_c^2)$.

The capital stock $K_{h,t}$ owned by household h evolves according to

$$K_{h,t} = (1 - \delta)K_{h,t-1} + e^{x_t^i} (1 - S(I_{h,t}/I_{h,t-1})) I_{h,t}, \quad (13)$$

where $S(I_t/I_{t-1})$ is the investment adjustment cost function with the following properties: $S(\gamma) = S'(\gamma) = 0$, and $S''(\gamma) = s'' > 0$. The investment-specific technological shock follows $x_t^i = \rho_i x_{t-1}^i + \varepsilon_t^i$, where $\varepsilon_t^i \sim \mathcal{N}(0, \sigma_i^2)$.

Households rent to firms an effective amount of capital $K_{h,t}^u$ given by

$$K_{h,t}^u = U_{h,t} K_{h,t-1}, \quad (14)$$

where $U_{h,t}$ is the level of capital utilization. They receive $R_t^k K_{h,t}^u$ for renting capital but pay a cost $a(U_{h,t})K_{h,t-1}$ in terms of the consumption good. This function has the properties $a(\bar{U}) = 0$ and $a''(\bar{U}) = a''$, where $\bar{U}$ is the capital utilization rate in the steady-state.

The households' budget constraint in each period t is given by

$$P_t(C_{h,t} + I_{h,t} + a(U_{h,t})K_{h,t-1}) + B_{h,t} \leq R_{t-1}B_{h,t-1} + W_{h,t}L_{h,t} + R_t^k U_{h,t}K_{h,t-1} + \Xi_t + T_{h,t}. \quad (15)$$

Ξ_t denotes firms' profits distributed to households and $T_{h,t}$ denotes net transfers from the government. Households trade only a nominal one-period risk-free bond $B_{h,t}$.⁹

Households supply labor in a market with monopolistic competition subject to a wage-setting friction. Following [Erceg et al. \(2000\)](#), in every period only a fraction $1 - \xi_w$ of

⁹Asset markets are incomplete: households cannot insure idiosyncratic wage and expectational risk through state-contingent claims, so consumption, investment, and capital holdings are heterogeneous across households. Habit formation is internal: the reference stock is the household's own past consumption, $C_{h,t-1}$. We nonetheless assume that households do not internalize the effect of current consumption on next period's habit stock. This behavioral friction keeps the Euler equation in its standard form and adds no further channel through which higher-order expectations operate. See [Appendix G](#) for the derivation.

households are able to optimize their wages. Those households choose their optimal wage by setting a mark-up over the marginal rate of substitution between consumption and labor. Households who are not able to optimize update their wage using an indexation rule given by

$$W_{h,t} = (\Pi_{t-1})^{\iota_w} (\bar{\Pi})^{1-\iota_w} \gamma W_{h,t-1}, \quad (16)$$

which adjusts wages by the steady-state real growth and a geometrically weighted average of steady-state inflation ($\bar{\Pi}$) and the previous period's inflation (Π_{t-1}).

When able to re-optimize, household h chooses $W_{h,t}^*$ to maximize expected discounted utility subject to the budget constraint (15) and the demand for its labor variety (see Appendix G for the full derivation). In future periods $t+s$, the nominal wage follows $W_{h,t+s} = X_{t,t+s}^w W_{h,t}^*$, where

$$X_{t,t+s}^w = \begin{cases} (\gamma \bar{\Pi}^{1-\iota_w})^s \Pi_{j=1}^s (\Pi_{t+j-1}^{\iota_w}) & \text{if } s \geq 1 \\ 1 & \text{if } s = 0. \end{cases} \quad (17)$$

Given optimal prices and the indexation rule, the aggregate wage level follows

$$W_t = \left[\xi_w \left(\gamma \Pi_{t-1}^{\iota_w} \bar{\Pi}^{1-\iota_w} W_{t-1} \right)^{-\frac{1}{\mu_t^w}} + (1 - \xi_w) \int_0^1 \left(W_{h,t}^* \right)^{-\frac{1}{\mu_t^w}} dh \right]^{-\mu_t^w}. \quad (18)$$

Government Policies

The central bank sets the nominal interest rate according to a standard Taylor rule with smoothing according to

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\phi_r} \left[\left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} \right]^{(1-\phi_r)} e^{x_t^r}, \quad (19)$$

where variables with a bar denote the steady-state values. The monetary policy shock follows $x_t^r = \rho_r x_{t-1}^r + \varepsilon_t^r$, where $\varepsilon_t^r \sim \mathcal{N}(0, \sigma_r^2)$. For simplicity, we assume that the monetary authority responds to deviations of output from its steady-state value instead of deviations from the natural output.

The government budget constraint is the following

$$P_t G_t + R_{t-1} B_{t-1} = B_t + T_t \quad (20)$$

where $T_t = \int_0^1 T_{h,t} dh$ and B_t are total lump-sum taxes and bonds, respectively. Government expenditure follows a simple stochastic process given by $G/Y = g_y + x_t^g$, where $g_y \equiv \bar{G}/\bar{Y}$ is the steady-state ratio of government expenditure to output and x_t^g is a government expen-

diture shock with process $x_t^g = \rho_g x_{t-1}^g + \varepsilon_t^g$, where $\varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2)$.

Resource constraint and market clearing

The aggregate resource constraint

$$C_t + I_t + G_t + a(U_t)K_{t-1} = Y_t, \quad (21)$$

is derived by integrating households' budget constraint over h , and combining it with the zero profit condition of final goods firms and labor packers and the government budget constraint.

Market clearing in labor, capital markets and government bonds holds such that $L_t = \int_0^1 L_{i,t} di$, $K_t^u = \int_0^1 K_{i,t} di$, and $B_t = \int_0^1 B_{h,t} dh$, respectively.

Information and signal extraction

Intermediate good firms and households do not perfectly observe the shocks driving the economy; instead, they receive noisy idiosyncratic signals about them. Formally, the signals follow

$$s_{j,t}^l = x_t^l + v_{j,t}^l, \quad v_{j,t}^l \sim \mathcal{N}(0, \tau_l^2), \quad (22)$$

where $l \in \{a, c, i, g, p, w, r\}$ denotes each type of shock and $j \in \{h, i\}$ is either a household h or firm i , both indexed on $[0, 1]$. This assumption implies that firms and households receive signals with the same properties and are subject to the same degree of informational frictions. Hence, an agent j 's information set is described as

$$\mathcal{I}_t^j = \{s_{j,\tau}^a, s_{j,\tau}^c, s_{j,\tau}^i, s_{j,\tau}^g, s_{j,\tau}^p, s_{j,\tau}^w, s_{j,\tau}^r : \tau \leq t\}. \quad (23)$$

Given that households and firms have the same unit mass, their average expectation is the same and denoted by $\bar{E}_t[\cdot] \equiv \int_0^1 E_{j,t}[\cdot] dj$, where $E_{j,t}[\cdot] \equiv E[\cdot | \mathcal{I}_t^j]$ denotes the j 's individual expectation. Given the AR(1) structure of the shocks and signal structure (22), j 's rational expectation about the shock x_t^l is computed by using the Kalman filter such that

$$E_{j,t}[x_t^l] = E_{j,t-1}[x_t^l] + \bar{k}_l [s_{j,t}^l - E_{j,t-1}[s_{j,t}^l]] \quad (24)$$

where $\bar{k}_l$ denotes the steady-state Kalman gain of shock l given by

$$\bar{k}_l = \frac{\tilde{p}_l(\rho_l, r_l)}{\tilde{p}_l(\rho_l, r_l) + r_l^2}, \quad (25)$$

where $r_l \equiv \frac{\tau_l}{\sigma_l}$ denotes the noise-to-signal ratio of signal l and $\tilde{p}_l(\rho_l, r_l)$ is defined as the ratio of the individual expectation steady-state variance and the shock variance. The latter can be computed using a standard Riccati equation and is a function of the l shock persistence and its noise-to-signal ratio (see Appendix B for details). Therefore, the Kalman gain, $\bar{k}_l = k(\rho_l, r_l)$, is the key object that measures the degree of informational friction, *conditional on the shock's parameters*.

The average expectation is given by

$$\bar{E}_t[x_t^l] = (1 - \bar{k}_l)\bar{E}_{t-1}[x_t^l] + \bar{k}_l x_t^l = (1 - \bar{k}_l)\rho_l \bar{E}_{t-1}[x_{t-1}^l] + \bar{k}_l x_t^l. \quad (26)$$

Thus, the current expectation is a weighted average between the past expectation and the actual shock, whose weight is given by the Kalman gain.

If $\bar{k}_l = 1$ ($\tau_l = 0$, $r_l = 0$) the model boils down to the full information benchmark. If $\bar{k}_l = 0$ ($\tau_l \rightarrow \infty$, $r_l \rightarrow \infty$), the average expectation is zero (the unconditional expectation).¹⁰

Equation (26) helps to understand why and when information frictions can generate inertia. Strong informational frictions (low $\bar{k}_l$) have two implications for average expectations: they dampen the response to the actual shock and increase the expectation persistence. The latter also depends on the persistence of the unobserved shock, ρ_l . For instance, the expectation has no persistence regardless of the degree of information friction when $\rho_l = 0$.

These properties are key for understanding shock propagation and whether informational frictions substitute for standard frictions as discussed in section 5.

2.1 Log-linearized equilibrium

For brevity, the log-linearized equations and detrending procedure are detailed in Appendix A. The derivation of these equations can be found in Appendix G through Appendix I. Hereafter, lowercase variables represent log-deviations from the steady state of their uppercase counterparts.

To clarify how dispersed information affects equilibrium conditions, we highlight two key equilibrium conditions. First, consider the Euler equation:

$$c_t = \frac{\varphi/\gamma}{1 + \varphi/\gamma} c_{t-1} + \frac{1}{1 + \varphi/\gamma} \int_0^1 E_{h,t}[c_{h,t+1}] dh - \frac{(1 - \varphi/\gamma)}{1 + \varphi/\gamma} \bar{E}_t[r_t - \pi_{t+1} + (x_{t+1}^c - x_t^c)].$$

This is the same Euler equation as in standard representative-agent models (Smets and Wouters, 2003) with three key differences due to dispersed information and incomplete markets.

¹⁰Zero is the solution for the equation $\bar{E}[x_t^l] = \rho_l \bar{E}_{t-1}[x_{t-1}^l]$.

First, expectations are heterogeneous, and hence average expectations replace the full-information expectation operator. Second, current endogenous variables and structural shocks are not observed. In the context of the households' intertemporal consumption decision, households must form expectations about the current nominal rate, r_t , and the current preference shock, x_t^c . Third, the forward-looking term is an integral of individual expectations about the household's *own* future consumption. Since the law of iterated expectations does not hold for average expectations (Morris and Shin, 2005), $\int_0^1 E_{h,t}[c_{h,t+1}] dh \neq \bar{E}_t[c_{t+1}]$. The same applies to investment, the shadow price of capital, and optimal prices and wages (see Appendix A), which together form the block multiplying F_1 in equation (29).

Second, consider the New Keynesian Phillips curve (NKPC) given by

$$\pi_t = \xi_p \iota_p \pi_{t-1} + \kappa_p \xi_p \bar{E}_t[mc_t + x_t^p] + \psi_p \bar{E}_t[\pi_t] + (1 - \xi_p) \beta \xi_p \int_0^1 E_{i,t}[p_{i,t+1}^*] di \quad (27)$$

where $\psi_p \equiv (1 - \xi_p)(1 - \iota_p \beta \xi_p)$.

The same three differences appear in the NKPC: firms form average expectations about current marginal cost and the price mark-up shock, and the forward term involves firms' expectations about their future *own optimal prices*, with $\int_0^1 E_{i,t}[p_{i,t+1}^*] di \neq \bar{E}_t[p_{t+1}^*]$.¹¹ The NKPC adds a fourth, distinctive feature: inflation depends on its own average expectation, $\bar{E}_t[\pi_t]$, with coefficient ψ_p . As pointed out by Nimark (2008), this self-reference is what makes higher-order expectations matter for inflation dynamics. By taking the average expectations of this equation and substituting it iteratively, we obtain that

$$\pi_t = \frac{\xi_p \iota_p}{1 - \psi_p} \pi_{t-1} + \kappa_p \xi_p \sum_{k=1}^{\infty} \psi_p^{k-1} \bar{E}_t^{(k)}[mc_t + x_t^p] + (1 - \xi_p) \beta \xi_p \sum_{k=0}^{\infty} \psi_p^k \bar{E}_t^{(k)} \left[\int_0^1 E_{i,t}[p_{i,t+1}^*] di \right], \quad (28)$$

where $\bar{E}_s^{(k)}[z_t] \equiv \int_0^1 E_{is}[\bar{E}_s^{(k-1)}[z_t]] di$, for any variable z_t , all $k \geq 1$ and $s \leq t$, with the convention that $\bar{E}_t^{(0)}[z_t] \equiv z_t$ and $\bar{E}_t^{(1)}[z_t] \equiv \bar{E}_t[z_t]$. The expansion makes explicit that firms' beliefs enter at every order, with the dependence on order k decaying geometrically at rate $\psi_p \in (0, 1)$. The same higher-order dependence with geometric decay applies to real wage dynamics (see equation I.12).

One key feature of the solution method discussed in the next section is that we do not need to represent equilibrium conditions in terms of higher-order expectations as in equation (28) to solve the model. We can find a policy function that depends on higher-order expectations even defining equilibrium conditions in the system of equations in terms of average expectations as in equation (27).

¹¹As pointed out in Appendix D of Angeletos and Huo (2021), Nimark (2008) and Melosi (2017) abstract that $\int_0^1 E_{i,t}[p_{i,t+1}^*] di \neq \bar{E}_t[p_{t+1}^*]$ in their derivation. This difference is key because it affects the definition of the general system of equilibrium conditions (29) and the guessed policy function (32).

3 Solution method

In this section, we present a novel solution method for DSGE models featuring imperfect and dispersed information.

The log-linearized equilibrium equations can be written as the following general system of linear equations

$$\begin{aligned} F_1 \int_0^1 E_{i,t}[Y_{i,t+1}]di + F_2 \bar{E}_t[Y_{t+1}] + G_1 Y_t + G_2 \bar{E}_t[Y_t] + H Y_{t-1} + \\ L \bar{E}_t[x_{t+1}] + M_1 x_t + M_2 \bar{E}_t[x_t] = 0_{m \times 1}, \end{aligned} \quad (29)$$

where $i \in [0, 1]$ indexes an agent, $Y_{i,t}$ is a $m \times 1$ vector of individual choices, Y_t is a $m \times 1$ vector of aggregate endogenous variables and x_t is a $n \times 1$ vector of unobservable exogenous shocks.

The system of equations (29) extends existing solution methods along two dimensions. First, it incorporates average expectations about future own variables (the term multiplying F_1). Second, it accommodates endogenous state variables (the term multiplying H), which is necessary for solving medium-scale DSGE models such as the one in Section 2.

Structural shocks x_t follow a stationary stochastic process

$$x_t = A_1 x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \Sigma_\varepsilon) \quad (30)$$

where A_1 is a diagonal matrix storing the persistence of each shock, and Σ_ε is a diagonal covariance matrix. Idiosyncratic signals are given by

$$s_{i,t} = C_x x_t + D v_{i,t}, \quad v_{i,t} \sim \mathcal{N}(0, \Sigma_v), \quad (31)$$

where Σ_v is a diagonal covariance matrix.¹²

Define the k -th order average expectation $E_t^{(k)}[\cdot]$ given information of period t about the vector of unobservable aggregate shocks x_t as $E_s^{(k)}[x_t] \equiv \int_0^1 E_{is} [E_s^{(k-1)}[x_t]] di$, for all $k \geq 1$ and s, t , with the convention that $E_t^{(0)}[x_t] \equiv x_t$ and $E_t^{(1)}[x_t] \equiv \bar{E}_t[x_t]$.

It is useful to denote the expectations hierarchy of x_t from order l to s as the vector

$$x_t^{(l:s)} \equiv \begin{bmatrix} E_t^{(l)}[x_t]' & E_t^{(l+1)}[x_t]' & \cdots & E_t^{(s)}[x_t]' \end{bmatrix}',$$

for $s > l \geq 0$.

¹²The model from Section 2 implies a diagonal structure for A_1 , Σ_ε and Σ_v . The solution method does not require these assumptions.

Building on the insights of [Nimark \(2008\)](#) and [Melosi \(2017\)](#), our solution method involves truncating the state space to address the issue of infinite regression of expectations, a problem first highlighted by [Townsend \(1983\)](#).¹³

Specifically, we guess that the individual and aggregate equilibrium law of motion and expectation hierarchy are given by

$$Y_{i,t} = \mathbf{R}Y_{i,t-1} + \mathbf{Q}_0x_t + \mathbf{Q}_1E_{i,t} \left[x_t^{(0:\bar{k})} \right], \quad (32)$$

$$Y_t = \mathbf{R}Y_{t-1} + \mathbf{Q}x_t^{(0:\bar{k})} \quad (33)$$

$$x_t^{(0:\bar{k})} = \mathbf{A}x_{t-1}^{(0:\bar{k})} + \mathbf{B}\varepsilon_t, \quad (34)$$

where $(\mathbf{R}, \mathbf{Q}_0, \mathbf{Q}_1, \mathbf{A}, \mathbf{B})$ are finite dimensional matrices to be determined in equilibrium. $\mathbf{Q} = \mathbf{Q}_0e_x + \mathbf{Q}_1T$, e_x is the selection matrix such that $x_t = e_x x_t^{(0:\bar{k})}$ and T is a order transformation matrix such that $E_t^{(1)} \left[x_t^{(0:\bar{k})} \right] = T x_t^{(0:\bar{k})}$.¹⁴

Our guess-and-verify strategy extends [Melosi \(2017\)](#) in two aspects: First, we guess the policy function of individuals (32) and then aggregate to find the aggregate policy function (33). In contrast, [Melosi \(2017\)](#) abstracts from average expectations of future own variables in the system of equations (specifically, the term multiplying F_1 in equation 29) and directly guesses the aggregate policy function.

Second, both the individual and aggregate policy functions include a lagged term with a coefficient $\mathbf{R}$. This is significant because our system incorporates endogenous state variables, such as Y_{t-1} in equation (29). By doing so, we allow endogenous variables to exhibit intrinsic persistence, in addition to the persistence arising from shocks and higher-order expectations.

Proposition 1 shows the dynamics for expectation hierarchy and the expressions for the fixed-point solution for $(\mathbf{A}, \mathbf{B})$.

Proposition 1. *Suppose that x_t is a stationary process, the expectation hierarchy follows equation (34) and agents use information from signals (31). Assuming common knowledge of rationality, the individual and average expectations about the hierarchy are given by*

$$\begin{aligned} E_{i,t} \left[x_t^{(0:\bar{k})} \right] &= (I_k - \mathbf{K}C) \mathbf{A}E_{i,t-1} \left[x_{t-1}^{(0:\bar{k})} \right] + \mathbf{K}C\mathbf{A}x_{t-1}^{(0:\bar{k})} + \mathbf{K}C\mathbf{B}\varepsilon_t + \mathbf{K}Dv_{it} \\ \bar{E}_t \left[x_t^{(0:\bar{k})} \right] &= (I_k - \bar{\mathbf{K}}C) \mathbf{A}\bar{E}_{t-1} \left[x_{t-1}^{(0:\bar{k})} \right] + \bar{\mathbf{K}}C\mathbf{A}x_{t-1}^{(0:\bar{k})} + \bar{\mathbf{K}}C\mathbf{B}\varepsilon_t. \end{aligned}$$

where $C = C_x e_x$.

¹³[Nimark \(2017\)](#) shows that if the impact of higher-order expectations on equilibrium outcomes diminishes with their order, there exists a threshold $\bar{k}$ at which the approximation error becomes arbitrarily small.

¹⁴See details in Appendix D.

The expectation hierarchy consistent with the expectations above has coefficients $(\mathbf{A}, \mathbf{B})$ that satisfy the matrix equations:

$$\begin{aligned} (I_k - T' \bar{\mathbf{K}} C) \mathbf{A} &= e'_x A_1 e_x + T' (I_k - \bar{\mathbf{K}} C) \mathbf{A} T \\ (I_k - T' \bar{\mathbf{K}} C) \mathbf{B} &= e'_x \end{aligned} \quad (35)$$

such that $\bar{\mathbf{K}}$ and $\bar{\mathbf{P}}$ solve the Riccati equation resulting from equations

$$\begin{aligned} \bar{\mathbf{K}} &= \bar{\mathbf{P}} C' [C \bar{\mathbf{P}} C' + D \Sigma_v D']^{-1} \\ \bar{\mathbf{P}} &= \mathbf{A} [\bar{\mathbf{P}} - \bar{\mathbf{K}} C \bar{\mathbf{P}}] \mathbf{A}' + \mathbf{B} \Sigma_\epsilon \mathbf{B}' \end{aligned} \quad (36)$$

The Riccati equation resulting from equations (36) solves the steady-state mean square error of the expectation hierarchy (see also Appendix B for the scalar case). The equations use the standard Kalman filter with hierarchy dynamics (34) as state equation and measurement equation as signals (31).

The key difference from a standard Kalman filter is that $(\mathbf{A}, \mathbf{B})$ are themselves determined by agents' signal extraction problem. Thus, the hierarchy persistence ($\mathbf{A}$) and response to shocks ($\mathbf{B}$) depend on the Kalman gain matrix $\bar{\mathbf{K}}$. As any signal extraction, the $\bar{\mathbf{K}}$ depends on the unobserved state dynamics $(\mathbf{A}, \mathbf{B})$. Therefore, the equilibrium solution $(\mathbf{A}, \mathbf{B}, \bar{\mathbf{K}}, \bar{\mathbf{P}})$ is the fixed point from equations (35-36).

Since the signals (31) are exogenous, they do not depend on equilibrium conditions (29). This simplifies finding the guessed coefficients for the equilibrium $(\mathbf{R}, \mathbf{Q}_0, \mathbf{Q}_1)$ and substantially reduces computational costs. In contrast, Nimark (2008) and Melosi (2017) allow for endogenous signals.¹⁵

Before presenting the equilibrium dynamics in Proposition 2 below, we want to emphasize one assumption that substantially simplifies this task. We assume that when forming expectations about Y_t using the guessed solutions (32-33), agents know the past value of endogenous variables, Y_{t-1} .

This assumption contrasts with the expectation formation from Proposition 1 in which expectations depend only on exogenous private signals. This avoids learning from (past) endogenous variables, which generates a feedback between the signal extraction and endogenous variable responses to the hierarchy.¹⁶ Ribeiro (2017) considers the general case with endogenous signals.

¹⁵For instance, Melosi (2017) explains in his replication files that: "Be aware that estimating the DIM may take several weeks or even a few months depending on the computer used for this task." In our application, which has a considerably larger-scale model, takes roughly 72 (36) hours for 400k posterior draws using the RB-RWHM (RWMH) algorithm in a standard notebook.

¹⁶One can think of this assumption as a deviation from rationality as agents do not incorporate that their

This assumption is implicit in standard solution methods for imperfect common information such as [Blanchard et al. \(2013\)](#) and [Baxter et al. \(2011\)](#). This is also a common assumption when studying the New Keynesian Phillips Curve under dispersed information (e.g., [Angeletos and La'O, 2009](#); [Angeletos and Huo, 2021](#)).

Proposition 2. *For a given expectation hierarchy from equation (34), the system of equations (29) has a recursive equilibrium law of motion (33) whose matrix coefficients satisfy:*

1. $(\mathbf{R}, \mathbf{Q}_0, \mathbf{Q}_1)$ satisfy the matrix equations:

$$F\mathbf{R}^2 + G\mathbf{R} + H = 0_{m \times m} \quad (37)$$

$$[F_1\mathbf{R} + G_1] \mathbf{Q}_0 + M_1 = 0_{m \times n} \quad (38)$$

$$[F_1\mathbf{R} + G_1] \mathbf{Q}_1 + F_1\mathbf{Q}_1\mathbf{A} + (F_2\mathbf{R} + G_2)\mathbf{Q}_1T + F_2\mathbf{Q}_1T\mathbf{A} + [(F_2\mathbf{R} + G_2)\mathbf{Q}_0 + F\mathbf{Q}_0A_1 + (LA_1 + M_2)] e_x = 0_{m \times k} \quad (39)$$

where $F \equiv F_1 + F_2$, $G \equiv G_1 + G_2$ and $k = n(\bar{k} + 1)$.

2. Following [Uhlig \(2001\)](#), $\mathbf{R}$ has a unique stable solution if all eigenvalues of $\mathbf{R}$ are smaller than unity in absolute value.
3. Given the solution of $\mathbf{R}$, denote the matrices (V_0, V_1) such that:

$$V_0 = F_1\mathbf{R} + G_1 \quad (40)$$

$$V_1 = I_k \otimes (F_1\mathbf{R} + G_1) + T' \otimes (F_2\mathbf{R} + G_2) + \mathbf{A}' \otimes F_1 + \mathbf{A}'T' \otimes F_2 \quad (41)$$

Provided that there exists an inverse for V_0 and V_1 , the equilibrium solution for $(\mathbf{Q}_0, \mathbf{Q}_1)$ is given by

$$\mathbf{Q}_0 = -V_0^{-1}M_1 \quad (42)$$

$$\text{vec}(\mathbf{Q}_1) = -V_1^{-1} \text{vec}([(F_2\mathbf{R} + G_2)\mathbf{Q}_0 + F\mathbf{Q}_0A_1 + (LA_1 + M_2)] e_x) \quad (43)$$

where $\text{vec}(\cdot)$ denotes columnwise vectorization.

Given the individual response to shocks $(\mathbf{Q}_0)$ and to the individual expectation about the hierarchy $(\mathbf{Q}_1)$, the aggregate response to the hierarchy is given by: $\mathbf{Q} = \mathbf{Q}_0e_x + \mathbf{Q}_1T$

Proposition 2 connects the solution of imperfect common knowledge models with the standard undetermined coefficients solution for full information DSGE models. Condition knowledge about Y_{t-1} can be used to learn about $x_{t-1}^{(0, \bar{k})}$, which would also change their beliefs about $x_t^{(0, \bar{k})}$.

(37) is exactly the same as the usual “brute force” approach of Uhlig (2001). In other words, the equilibrium $\mathbf{R}$ is the same that would happen if agents had full information, as shown in Appendix D.¹⁷ Note that when we abstract from state endogenous variables ($H = 0_{m \times m}$), $\mathbf{R} = 0_{m \times m}$, leading to a policy function without intrinsic persistence from endogenous variable as in Melosi (2017).

The fact that $\mathbf{R}$ is invariant to the information structure has two implications. First, $\mathbf{R}$ can be computed with standard full-information techniques. Second, informational frictions do not affect how endogenous variables respond to lagged endogenous variables. Persistence of endogenous variables is still shaped by information frictions, however, through their effect on the expectation hierarchy $x_t^{(0:\bar{k})}$.

Appendix D presents the proofs for Propositions 1 and 2 and shows the details of the algorithm to find the fixed point solution from Proposition 1.

4 Estimation

In this section, we discuss the data, the prior choices, and the posterior results of the Bayesian estimation. We estimate the model featuring dispersed information (‘DI model’) using two different datasets: one containing macroeconomic data only and another including both macroeconomic and expectations data. For comparison, we also estimate the same model under full-information (‘FI model’) in the same datasets.

4.1 Data

We collect U.S. quarterly macroeconomic and expectations data from 4Q:1981 to 4Q:2007.¹⁸ The data on macroeconomic aggregates include the series of real GDP, real consumption, real investment, GDP deflator index, annualized 3-month Treasury bill rate, hours worked index and nominal wages index, and come from the Federal Reserve Database (FRED).¹⁹ We also gather the average expectation about current and one-quarter-ahead real GDP, real consumption and real investment, inflation and three-month Treasury bill rate from the U.S. Survey of Professional Forecasters (SPF). A detailed description of the datasets can be found in Appendix C.

When estimating medium-scale DSGE models, the inclusion of expectations data is rare.

¹⁷The key assumption for this result is that agents observe past endogenous variables at period t .

¹⁸We restrict our sample to avoid the Zero Lower Bound on nominal interest rates. Including non-linearities in the model complicates substantially the computation of expectations under imperfect information.

¹⁹The nominal interest rate is measured by 3-month Treasury bill rate to be consistent with the expectations data, as the SPF does not survey forecasts for the Federal funds rate.

In recent years, few papers have tried to incorporate expectations data to discipline macroeconomic models with complete and incomplete information. For instance, [Del Negro and Eusepi \(2011\)](#) adds one-year ahead inflation expectations to estimate a conventional New Keynesian model. [Melosi \(2017\)](#) estimates a small scale dispersed information model using data on inflation forecasts one-quarter and one-year ahead. [Del Negro et al. \(2015\)](#) uses ten-year inflation expectations from the Blue Chip and SPF.

We depart from previous literature in two ways. First, we use expectations data on a broader set of variables. Second, we use data on *current forecast revisions* given by

$$\overline{Rev}_{t|t-1}[Z_t] = \bar{E}_t[Z_t] - \bar{E}_{t-1}[Z_t],$$

where $\overline{Rev}_{t|s}[Z_t]$ is the average expectation revision at period t of a forecast about Z_t made in period s , and $\bar{E}_s[Z_t]$ is the average expectation about Z_t given information of period s .

Forecast revisions are particularly informative about information frictions. Aggregating equation (24) shows that

$$\overline{Rev}_{t|t-1}[x_t^l] \equiv \bar{E}_t[x_t^l] - \bar{E}_{t-1}[x_t^l] = \bar{k}_l [s_t^l - \bar{E}_{t-1}[s_t^l]] = \bar{k}_l \overline{FE}_{t-1}[x_t^l] \quad (44)$$

where $s_t^l = \int_0^1 s_{it}^l di = x_t^l$ is the aggregate signal, which due to the simple signal structure (22) is equal to the shock l . $\overline{FE}_s[x_t^l] = x_t^l - \bar{E}_s[x_t^l]$ denotes the average forecast error about x_t^l at period t of a forecast made in period s .

Note that the coefficient of the forecast error in the current revision is the Kalman gain $\bar{k}_l$ alone, isolating the information friction. In contrast, by iterating equation (26), one can see that the forecast $\bar{E}_t[x_t^l]$ depends on the whole history of previous signals, the information frictions and the shock's persistence. Moreover, longer forecast horizons ($\bar{E}_t[x_{t+h}^l]$) and revisions ($\overline{Rev}_{t|t-1}[x_{t+h}^l]$) are likely to be less informative as they depend also on the shock's persistence.²⁰

The clean mapping between revisions and Kalman gains in equation (44) holds for exogenous shocks. In practice, we observe endogenous variables, whose forecast revisions also reflect other sources of persistence. Nevertheless, short-horizon forecast revisions remain the most informative observable for identifying information frictions, as they minimize the confounding role of shock persistence.

We map macroeconomic data to model variables following the same approach as in SW. Specifically, real output, consumption, investment (per capita) and real wages are linked to the model in growth rates. Per capita total hours are demeaned while inflation and nominal

²⁰The h -step ahead forecast is given by $\bar{E}_t[x_{t+h}^l] = \rho_l^h \bar{E}_t[x_t^l]$ and analogous revision is $\overline{Rev}_{t|t-1}[x_{t+h}^l] = \rho_l^h \overline{Rev}_{t|t-1}[x_t^l]$. They also depend on shocks' persistence, ρ_l and the horizon h .

rates are measured in annual terms. We include measurement errors in the observational equations associated with current forecast revisions to avoid stochastic singularity²¹ and to capture the fact that the expectations data from SPF are not completely consistent with the average expectations of the agents populating our model.²²

The measurement equations that link data series with their model counterparts are presented in Appendix C.1.1.

4.2 Prior distributions

As standard in the literature, we fix the value of some parameters that are not easily identified and estimate the others. The quarterly depreciation rate δ is set to 2.5%. The ratio of government expenditures to output g_y is calibrated to its historical mean value of 18%. The steady-state wage markup $\bar{\mu}^w$ is fixed at 1.5 as in SW. Finally, the capital share of the production function α is fixed at 0.3 to be consistent with the historical capital-output ratio.

The introduction of informational frictions is likely to affect the estimation of real and nominal frictions, so we do not rely completely on estimates from previous papers to choose priors. Instead, most of the priors are relatively loose. Table 1 shows the choices for priors on the left side. The priors are the same for the FI and DI models. By definition, the full information model does not include the informational friction parameters and estimates with macroeconomic data only do not include measurement errors for revision data.

The prior for parameters that have support between 0 and 1, such as the persistence of shocks, partial adjustment of the Taylor rule, indexation of prices and wages to past inflation and habit persistence in consumption, follow a beta distribution ($\mathcal{B}$) with a mean of 0.5 and standard deviation of 0.2. One important exception is the Calvo frictions on prices and wages. For those two parameters we use very tight priors around micro data estimates for prices from Nakamura and Steinsson (2008) and for wages from Grigsby et al. (2021). Those imply an average duration of three quarters for prices and one year for wages.

The trend growth, inflation rate, and nominal interest rate steady-state parameters have priors with gamma distribution ($\mathcal{G}$) whose mean equals their sample average. The standard deviations are 0.10 for the first prior and 0.50 for the others.

The priors on the adjustment cost of investment, capital utilization, and the inverse of Frisch elasticity are taken from Del Negro et al. (2007). The parameters specifying the Taylor rule have priors with gamma distribution whose mean values are $\phi_\pi = 1.5$ and $\phi_y = 0.2$ and the standard deviations are 0.5 and 0.1, respectively.

²¹This is required since the complete dataset includes 12 time series, while the model has only 7 shocks.

²²This also can help to account the fact that data on macroeconomic aggregates are on constant chain level while the base chain in expectations change over time. See details in the data appendix (Appendix C).

Priors for the standard deviation of the structural shocks are distributed as an inverse gamma ($\mathcal{IG}$) with means chosen to match the relative importance of standard deviations of observables in the pre-sample 1957Q1-1981Q1, following a similar approach to [Del Negro and Eusepi \(2011\)](#). Standard deviations are set to 1.00, which implies fairly loose priors.

Priors for Kalman gains of each shock are set to follow a beta distribution with a mean of 0.5 and a standard deviation of 0.2. The value of 0.5 is similar to estimates from CG using one-year inflation expectations and [Bordalo et al. \(2020\)](#) for one-year data on output, consumption and investment growth rates, inflation, and nominal rates. Note that their estimates of frictions are using macroeconomic aggregates and not actual shocks. We discuss this further in [Section 5.1](#).

We use the Kalman gain ($\bar{k}_l$) directly as a parameter instead of the noise-to-signal ratio ($r_l = \tau_l/\sigma_l$) as [Del Negro and Eusepi \(2011\)](#) or the private signal variances (τ_l) as [Melosi \(2017\)](#). As discussed by [Del Negro and Eusepi \(2011\)](#), forming priors on r_l of a shock implicitly forms a prior for the private signal’s variance conditional on shock’s variance, σ_l . Analogously, when placing priors on the Kalman gains of each shock, it implicitly generates a prior for the NSR conditional on the shock’s persistence, ρ_l (see [equation 25](#)).

Therefore, despite the shock processes (ρ_l, σ_l) having different priors for each shock l , when assuming the same prior for the Kalman gains of all shocks, there is an implicit prior for $\tau_l|\rho_l, \sigma_l$ that allows the same information friction for all shocks a priori.

Finally, priors for the standard deviation of measurement errors on the time series of forecast revisions are distributed as inverse gamma with a mean that matches roughly 10% of the standard deviation of each revision data. The prior standard deviations are small to ensure very tight priors, which prevents measurement errors from explaining more than 25% of the corresponding standard deviation.

4.3 Posterior distributions

We estimate the models by combining the prior densities described above with the likelihood function computed using the Kalman filter. We solve the model truncating the hierarchy of average higher-order expectations at the sixth order, i.e., $\bar{k} = 6$.²³ We use the Random Block Random-Walk Metropolis-Hastings (RB-RWMH) algorithm with five blocks to draw from the posterior distribution.²⁴ [Table 1](#) reports the posterior moments for FI and DI models

²³[Nimark \(2017\)](#) shows that $\bar{k} = 6$ is an accurate approximation in a simple model. Which order is sufficiently high to provide a good approximation to the policy function is model dependent. We find similar results with $k = 12$ using the complete dataset and report them in [Appendix F](#). Truncating at lower orders substantially speeds up estimation.

²⁴Estimates from RB-RWMH have substantially better chain convergence diagnostics than standard RWMH as they generate chains with lower autocorrelation. We use 400k draws and a 50% burn-in. The

across both datasets.

The DI model estimated on the full dataset exhibits the following properties. We find strong information frictions across all shocks. The highest Kalman gain, 0.72, belongs to the monetary policy shock, which stands apart from the rest even though it remains considerably below full information. For all other shocks, posterior modes for the Kalman gains range from 0.01 to 0.52. The posteriors are precisely estimated, with the widest 90% credible interval reaching 0.3.

The posterior mode of habit formation is near unity across all four specifications, which is higher than the standard range of 0.7–0.8. A likely reason is that the standard practice is to impose a tight prior centered at 0.7, whereas we adopt a looser prior centered at 0.5.

The estimated degree of investment adjustment costs (IAC), 4.83, is slightly lower than SW’s original estimate of 5.48, although it remains noticeably higher than the 2.65 reported by [Justiniano et al. \(2011\)](#), who use the same investment series (including inventory investment) but only macroeconomic data.²⁵ The results for the FI model reinforce this pattern: including expectations data likewise drives higher IAC estimates. Turning to the nominal rigidities, estimated price indexation is negligible, while wage indexation is 0.76, exceeding the prior mean. The Taylor-rule reaction to inflation is high, with a posterior mode of 3.02, possibly reflecting that our sample excludes the pre-Volcker period because expectations data are unavailable for earlier years.

Finally, shock persistence varies substantially: investment-specific technology, preference, wage-markup, and monetary shocks are mostly transitory, whereas government spending and TFP shocks are highly persistent and price-markup shocks are moderately persistent.

Several additional results emerge from comparing datasets and model specifications. Expectations data are informative about information frictions: including them substantially changes the estimated Kalman gains. For investment, price-markup, TFP, and preference shocks, the Kalman gain falls sharply (implying higher frictions), while for the monetary shock it rises substantially (implying lower frictions). Regarding the standard inertial frictions, two parameters move substantially when expectations data are included. The IAC parameter rises by a factor of more than two, and even more so under FI. Wage indexation under DI also rises markedly, from 0.44 to 0.76, whereas it is essentially unchanged under FI.

candidate distribution is a multivariate normal with covariance matrix based on the estimate of the Hessian matrix at posterior mode as in [An and Schorfheide \(2007\)](#). Standard univariate convergence diagnostics suggest convergence for all models. For the DI model under the complete dataset, we also simulate three additional chains with different initial values based on draws of the prior and find that they converge to the same distribution.

²⁵SW use an investment series that excludes inventory investment; [Justiniano et al. \(2011\)](#) shows that this difference in the investment series drives the discrepancy.

Table 1: Prior and Posterior distributions

Parameters	Prior			Dataset: Macroeconomic data only						Dataset: Macroeconomic and expectations data					
				Posterior: FI model			Posterior: DI model			Posterior: FI model			Posterior: DI model		
	Dist.	Mean	Std. Dev.	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
<i>Endogenous propagation parameters</i>															
χ	$\mathcal{G}$	2.00	0.75	2.76	1.85	4.36	2.10	1.51	3.82	4.13	3.08	6.22	1.51	0.92	2.14
φ	$\mathcal{B}$	0.50	0.20	0.97	0.88	0.98	0.98	0.88	0.99	0.99	0.98	0.99	0.998	0.99	0.999
a''	$\mathcal{G}$	0.20	0.10	0.38	0.26	0.64	0.34	0.22	0.59	0.33	0.22	0.57	0.06	0.06	0.22
s''	$\mathcal{G}$	4.00	1.50	1.93	1.23	3.47	1.81	0.98	3.56	6.38	4.82	8.04	4.83	3.62	6.80
ϕ_y	$\mathcal{G}$	0.20	0.10	0.07	0.04	0.15	0.09	0.05	0.19	0.04	0.02	0.09	0.06	0.04	0.11
ϕ_π	$\mathcal{G}$	1.50	0.50	3.01	2.36	3.97	2.57	2.01	3.47	3.29	2.81	4.11	3.02	2.43	3.88
ϕ_r	$\mathcal{B}$	0.50	0.20	0.87	0.81	0.90	0.86	0.81	0.90	0.87	0.83	0.90	0.87	0.84	0.90
ι_p	$\mathcal{B}$	0.50	0.20	0.19	0.08	0.44	0.11	0.04	0.32	0.03	0.01	0.11	0.03	0.01	0.13
ι_w	$\mathcal{B}$	0.50	0.20	0.52	0.22	0.83	0.44	0.17	0.78	0.56	0.23	0.84	0.76	0.37	0.91
ξ_p	$\mathcal{B}$	0.67	0.05	0.83	0.79	0.86	0.81	0.76	0.84	0.83	0.81	0.85	0.84	0.80	0.87
ξ_w	$\mathcal{B}$	0.75	0.05	0.78	0.74	0.82	0.78	0.73	0.82	0.82	0.78	0.84	0.81	0.78	0.85
<i>Steady-state parameters</i>															
π_{ss}	$\mathcal{G}$	2.62	0.50	2.33	1.68	2.95	1.95	1.39	2.73	2.88	2.55	3.30	2.07	1.56	2.72
r_{ss}	$\mathcal{G}$	5.15	0.50	6.12	5.39	6.98	6.53	5.65	7.66	6.23	5.58	7.04	6.58	5.83	7.62
γ_{ss}	$\mathcal{G}$	0.50	0.10	0.31	0.25	0.37	0.31	0.25	0.36	0.34	0.30	0.37	0.36	0.31	0.41
<i>Structural shocks parameters</i>															
ρ_c	$\mathcal{B}$	0.50	0.20	0.36	0.21	0.78	0.23	0.14	0.55	0.24	0.15	0.33	0.14	0.07	0.21
ρ_i	$\mathcal{B}$	0.50	0.20	0.91	0.64	0.95	0.72	0.57	0.91	0.40	0.33	0.48	0.42	0.33	0.50
ρ_g	$\mathcal{B}$	0.50	0.20	0.98	0.96	0.99	0.98	0.97	0.99	0.97	0.96	0.98	0.97	0.95	0.99
ρ_p	$\mathcal{B}$	0.50	0.20	0.92	0.87	0.95	0.93	0.87	0.96	0.91	0.89	0.93	0.74	0.65	0.81
ρ_w	$\mathcal{B}$	0.50	0.20	0.19	0.08	0.36	0.14	0.06	0.25	0.21	0.09	0.34	0.17	0.09	0.25
ρ_r	$\mathcal{B}$	0.50	0.20	0.54	0.39	0.68	0.75	0.52	0.90	0.37	0.28	0.47	0.44	0.31	0.57
ρ_a	$\mathcal{B}$	0.50	0.20	0.95	0.87	0.98	0.94	0.87	0.98	0.87	0.82	0.92	0.999	0.998	0.9996
σ_c	$\mathcal{IG}$	0.50	1.00	0.27	0.25	0.35	0.34	0.29	0.63	0.27	0.24	0.30	1.06	0.87	1.52
σ_i	$\mathcal{IG}$	2.00	1.00	1.04	0.89	1.55	1.25	1.03	1.80	1.16	1.01	1.33	2.73	2.37	3.26
σ_g	$\mathcal{IG}$	0.20	1.00	0.57	0.49	0.65	0.52	0.45	0.63	0.59	0.53	0.69	0.55	0.49	0.65
σ_p	$\mathcal{IG}$	0.20	1.00	0.07	0.06	0.10	0.09	0.07	0.15	0.07	0.06	0.08	0.26	0.20	0.41
σ_w	$\mathcal{IG}$	0.50	1.00	0.35	0.29	0.41	0.58	0.44	1.50	0.34	0.29	0.41	0.94	0.72	1.76
σ_r	$\mathcal{IG}$	0.20	1.00	0.13	0.11	0.15	0.12	0.11	0.15	0.13	0.12	0.15	0.13	0.12	0.15
σ_a	$\mathcal{IG}$	0.20	1.00	0.41	0.38	0.48	0.41	0.37	0.47	0.46	0.41	0.53	0.47	0.42	0.54
<i>Information friction parameters^a</i>															
$\bar{k}_c$	$\mathcal{B}$	0.50	0.20				0.73	0.43	0.92				0.23	0.17	0.29
$\bar{k}_i$	$\mathcal{B}$	0.50	0.20				0.92	0.77	0.97				0.46	0.42	0.51
$\bar{k}_g$	$\mathcal{B}$	0.50	0.20				0.68	0.35	0.89				0.52	0.49	0.56
$\bar{k}_p$	$\mathcal{B}$	0.50	0.20				0.90	0.69	0.96				0.33	0.25	0.42
$\bar{k}_w$	$\mathcal{B}$	0.50	0.20				0.49	0.26	0.80				0.37	0.23	0.53
$\bar{k}_r$	$\mathcal{B}$	0.50	0.20				0.08	0.03	0.55				0.72	0.59	0.82
$\bar{k}_a$	$\mathcal{B}$	0.50	0.20				0.59	0.26	0.83				0.01	0.00	0.01
<i>Measurement errors standard deviation^b</i>															
σ_{dy}^{me}	$\mathcal{IG}$	0.10	0.02							0.52	0.46	0.59	0.30	0.26	0.34
σ_{dc}^{me}	$\mathcal{IG}$	0.07	0.02							0.43	0.39	0.48	0.20	0.19	0.23
σ_{di}^{me}	$\mathcal{IG}$	0.50	0.10							2.15	1.95	2.42	1.24	1.11	1.39
σ_{π}^{me}	$\mathcal{IG}$	0.15	0.04							0.77	0.69	0.87	0.51	0.46	0.59
σ_r^{me}	$\mathcal{IG}$	0.15	0.04							0.31	0.28	0.35	0.29	0.26	0.35
Marginal Likelihood				-956.03			-950.02			-1600.49			-1381.55		
Number of MH Draws				400k			400k			400k			800k		
MH Burn-in				50%			50%			50%			50%		

Note: $\mathcal{G}$, $\mathcal{IG}$ and $\mathcal{B}$ stand for: Gamma, Inverse Gamma and Beta distributions, respectively. ^aApplies to the dispersed information model. ^bApplies to the dataset including macroeconomic and expectations data.

Including expectations data also reduces the estimated persistence of preference, monetary, price-markup, and especially investment shocks,²⁶ while leaving the persistence of gov-

²⁶Posterior-mode estimates for investment persistence fall from 0.91 to 0.40 in the FI model and from 0.72 to 0.42 in the DI model.

ernment spending and wage-markup shocks largely unchanged. TFP persistence responds differently across models: under DI it rises close to unity, whereas under FI it falls slightly, remaining close to the macroeconomic-data estimate. Moreover, in the DI model, the shock standard deviations increase markedly for investment, price- and wage-markup, and preference shocks. As we show later, this increase is offset by stronger information frictions, mitigating the impact of these shocks on expectations.

Comparing the FI and DI models estimated on the full dataset further indicates that the FI model relies more heavily on IAC than the DI model to match the data. We also find a sizable reduction in the standard deviation of measurement errors, which provides an initial indication that the DI model better explains expectations data. This interpretation is confirmed by the marginal likelihood comparison reported at the bottom of Table 1: the advantage of the DI model rises substantially, reaching 219 log points. The next section investigates the quantitative implications of these estimates.

5 Business cycle dynamics under dispersed information

This section is divided into four parts. Section 5.1 explores whether simulated data from the estimated model reproduce empirical measures of information frictions. Sections 5.2 and 5.3 revisit business-cycle drivers and shock propagation in the DI model disciplined by macroeconomic and expectations data. Finally, Section 5.4 assesses the relative importance of standard inertial frictions and information frictions.

5.1 Matching empirical measures of information frictions

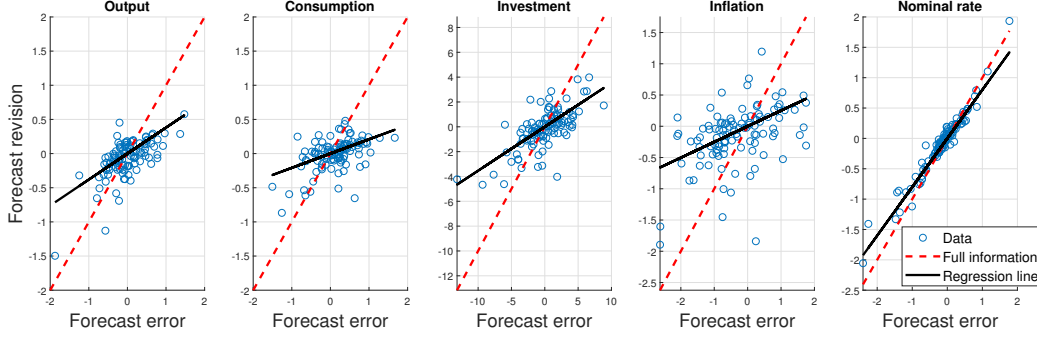
A growing literature following CG explores a relationship between forecast errors and forecast revisions implied by models with information frictions to obtain survey-based measures of those frictions. We adopt a similar empirical strategy to assess whether our estimated model replicates these stylized facts. For each variable z , we estimate

$$\overline{Rev}_{t|t-1}[z_t] = \beta \overline{FE}_{t-1}[z_t] + error_t. \quad (45)$$

According to equation (44), if z follows an AR(1) and agents form rational expectations with private dispersed information, then β is an estimate of the true Kalman gain, $\bar{k}_z$.

Figure 1 shows the scatter plot with the fitted line in solid black for each variable in the SPF. Hereafter, we refer to those estimates as empirical Kalman gains (eKGs).

Figure 1: Empirical Kalman gains from SPF expectations data (1981Q4-2007Q4)



Note: Empirical Kalman gains are the coefficients from regression (45). Output, consumption and investment refer to their respective *quarterly growth rate*, whereas inflation and interest rates refer to their *levels*.

A natural benchmark is full information, in which revisions and forecast errors line up one-for-one (the 45° line). Estimates depart markedly from this benchmark, and the degree of departure varies across variables, as documented by CG and [Bordalo et al. \(2020\)](#).

A caveat is that the measure from equation (45) truly holds only for the case of exogenous AR(1) fundamentals, whereas we observe endogenous variables whose equilibrium dynamics depend on lags and the hierarchy of expectations. As a result, eKGs generally reflect the interaction of information frictions with other sources of persistence in the model. Rather than treating eKGs as direct estimates of structural Kalman gains, we use them as an unmatched moment for external validation of the *model-implied* joint mapping from structural frictions to observable forecast revisions and errors.

We proceed as follows. For a given parameter vector Θ , we simulate $s = 1, \dots, n_{sim}$ samples of length T_{sim} from the model, construct simulated forecast revisions and forecast errors consistent with the measurement system used in estimation such that

$$\begin{aligned}\overline{Rev}_{z,t|t-1}^{(s)} &= \bar{E}_t[z_t]^{(s)} - \bar{E}_{t-1}[z_t]^{(s)} + \varepsilon_{z,t}^{me,(s)}, \\ \overline{FE}_{z,t-1}^{(s)} &= z_t^{(s)} - \bar{E}_{t-1}[z_t]^{(s)},\end{aligned}$$

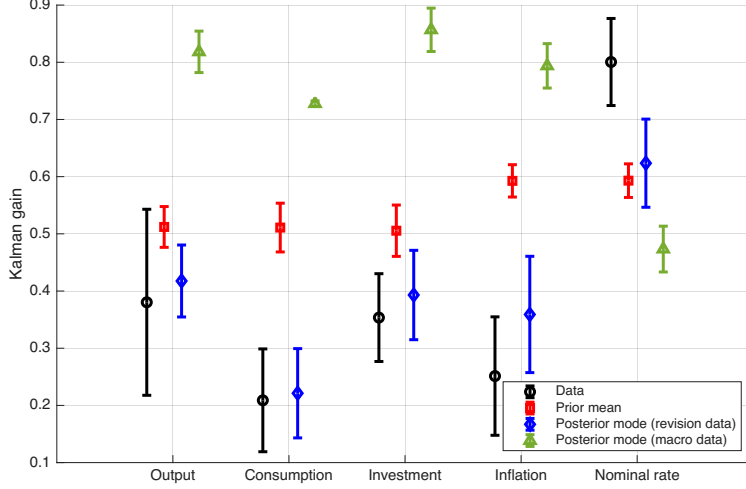
and estimate (45) on each simulated sample, s .

Figure 2 reports average coefficients and 90% confidence intervals across $n_{sim} = 100$ simulations with $T_{sim} = 105$, matching the SPF sample size.²⁷

Two results stand out. First, simulated eKGs under the *prior mean* (red $\square$ error bars)

²⁷We add simulated measurement errors $\varepsilon_{z,t}^{me}$ to forecast revisions so that the simulated objects are consistent with the measurement equations (C.1). $n_{sim} = 100$ is sufficiently large to avoid sampling variability affecting the Kalman gain estimates meaningfully.

Figure 2: Empirical Kalman gains from simulated baseline model (1981Q4-2007Q4)



Note: Empirical Kalman gains are the coefficients from regression (45) using actual and simulated data from the model with respective parameter configuration. Output, consumption and investment refer to their respective *quarterly growth rate*, whereas inflation and interest rates refer to their *levels*. Bars correspond to 90% confidence intervals.

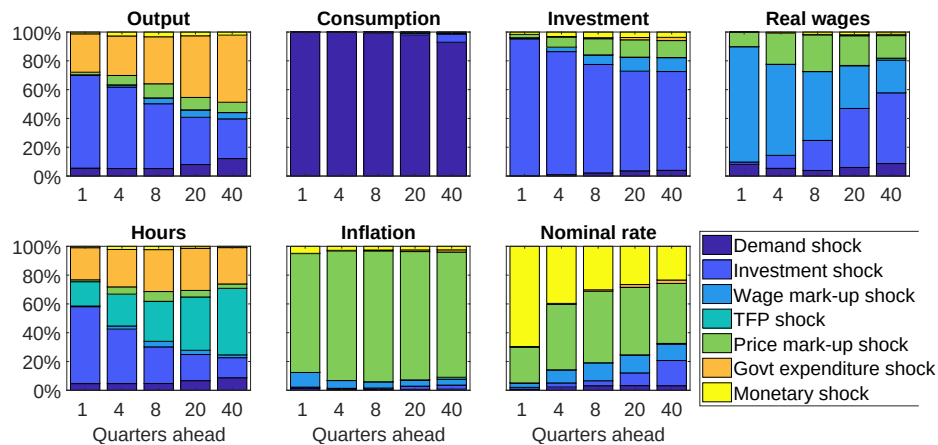
cluster mechanically around 0.5, reflecting the common prior mean on the Kalman gains of exogenous shocks. Second, simulated eKGs under the *posterior mode* estimated with the complete dataset (blue $\diamond$ error bars) align closely with the SPF eKGs, although the estimation does not directly target these coefficients. By contrast, the posterior mode from the model estimated with macroeconomic data only (green $\triangle$ error bars) performs starkly worse. Together, these findings highlight the importance of including expectations data in the model estimation and provide external validation that the model captures the learning dynamics embedded in survey forecast revisions.

5.2 Business cycle drivers

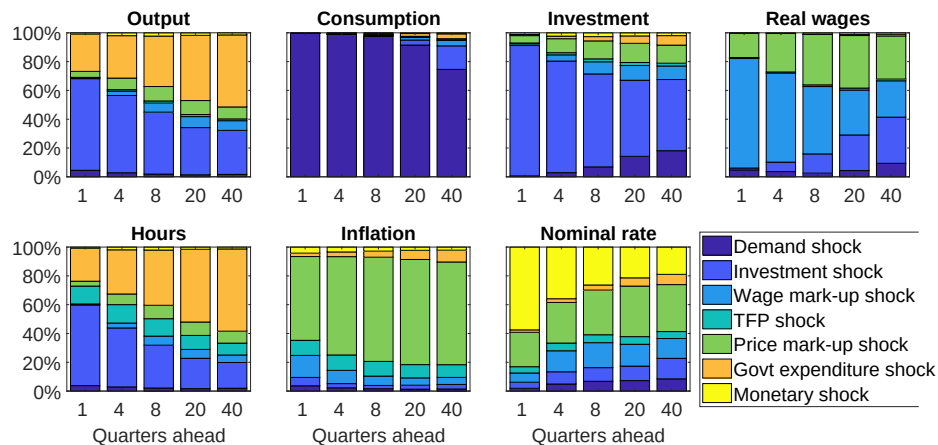
Having established that the model successfully replicates empirical measures of information frictions when disciplined by expectations data, we turn to business cycle drivers. We assess them through a forecast error variance decomposition (FEVD) exercise for selected variables at horizons ranging from 1 to 40 quarters.

Figure 3 reports results for both models estimated on the complete dataset. Investment-specific technology shocks and government expenditure shocks are the main drivers of output growth and hours worked. At short horizons, the investment shock accounts for roughly 60% of fluctuations, while government spending explains about 25%. At longer horizons, their relative importance reverses. This pattern is robust for both models. In contrast to SW, we

Figure 3: FEVD for macroeconomic variables for both models in the complete dataset



(a) DI model



(b) FI model

Note: Bars show the posterior mean of the FEVD at each horizon. All variables are in levels.

find a smaller role for wage mark-up and TFP shocks, and a larger role for investment-specific technology shocks. Our results align with the conclusions of [Justiniano et al. \(2011\)](#), who emphasize the importance of including inventories in total investment, which SW abstracts from. [Auclert et al. \(2020\)](#) also find that investment-specific technological shocks are key to output fluctuations using an estimated HANK model with sticky information.

Across variables, the dominant shock is typically the one that most directly shifts the corresponding equilibrium condition: preference shocks for consumption, investment shocks for investment, price mark-up shocks for inflation, and wage mark-up shocks for real wages. Beyond their dominant role for inflation, price mark-up shocks also contribute significantly

to nominal rate and real wage dynamics, with importance rising over time.²⁸

Comparing Panels (a-b) reveals that this pattern is more pronounced under DI. The underlying mechanism is the anchoring of expectations to prior beliefs: agents gradually update their beliefs about shocks, dampening general equilibrium propagation and amplifying the relative importance of partial equilibrium channels. In Section 5.3, we show that this attenuation is driven by the underreaction of average expectations.

5.3 Propagation of shocks

We compare Bayesian impulse response functions (IRFs) from the DI and FI models estimated on the complete dataset to assess how informational frictions affect shock transmission.

Learning and higher-order average expectations about exogenous shocks. Figure 4 displays the impulse response of each structural shock alongside the corresponding hierarchy of average expectations (first- through third-order). Under full information, expectations at all orders coincide with the true shock; systematic deviations from this benchmark quantify the gradual learning induced by information frictions. The DI and FI models are estimated separately, so their impulse responses also differ due to changes in the estimated exogenous processes.

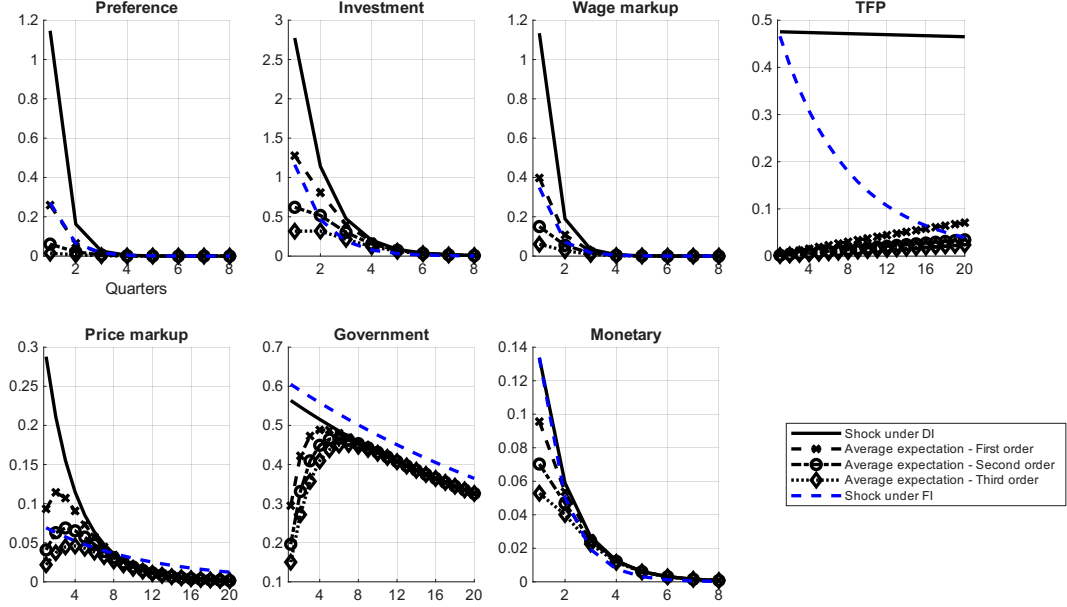
Two patterns emerge from the DI model. First, shocks estimated to have low persistence (preference, investment-specific technology, wage mark-up) generate negligible expectation inertia even with substantial information frictions. Agents underestimate the contemporaneous realization of these shocks, but because the shocks are transitory, expectations simply decay back toward the steady state alongside the shock itself. Learning therefore largely affects the impact response rather than producing sustained belief dynamics. To reproduce similar volatility in observables despite this attenuation, the DI model requires larger innovation variances than the FI model for these shocks.

Second, shocks that combine high persistence with strong information frictions (government spending, price mark-up) generate inertial, hump-shaped dynamics in first- and higher-order expectations. Both ingredients are needed: shock persistence propagates to the belief hierarchy, while strong information frictions slow the convergence of beliefs toward the true state.²⁹

²⁸TFP shocks play a minor role in explaining fluctuations in all observables.

²⁹TFP is an extreme case, with $\bar{k}_a$ very close to zero and ρ_a very close to one, which yields extremely slow higher-order expectation dynamics. This shock, however, accounts for a small share of the fluctuations in all observables, except hours in the longer run (see Figure 3). The opposite is true for the monetary policy shock, which combines low persistence with the weakest information friction of any shock.

Figure 4: Estimated learning process for each shock



Note: The figure shows the posterior mean of impulse responses for each shock, triggered by a one standard deviation shock, alongside the estimated responses of higher-order beliefs about the shock.

Both patterns are explained by equation (26). For each shock l , $\bar{k}_l$ controls how the average expectations react to signals and thus to shocks. The persistence of average expectations has two sources: it inherits the shock's own persistence, and it has an additional intrinsic component from gradual learning, governed by $(1 - \bar{k}_l)\rho_l$. Generating prolonged hump-shaped dynamics in average expectations therefore requires a combination of an attenuated response to shocks and high intrinsic persistence (low $\bar{k}_l$ and high ρ_l).

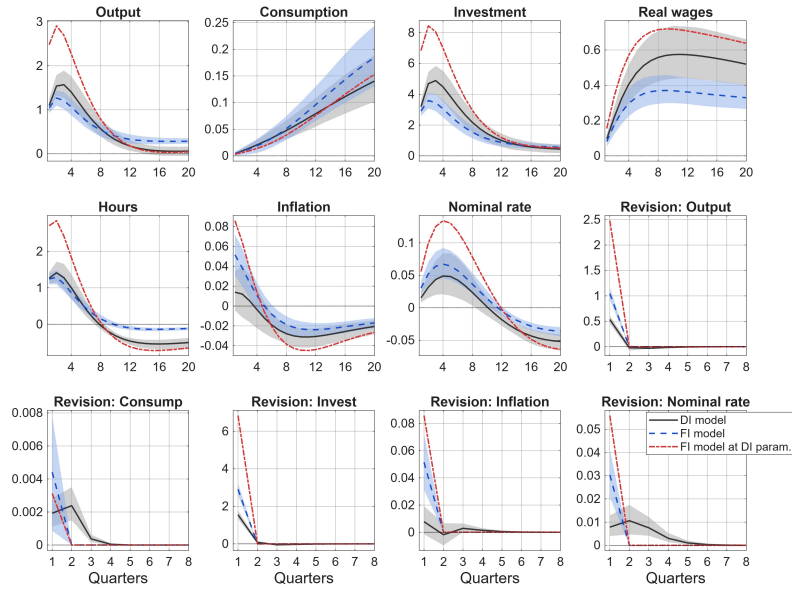
Quantitative explorations with small models typically calibrate shocks to be persistent and subject to strong information frictions. Our estimates support this configuration for some shocks, but not all. For a number of shocks, the data favor a combination of low persistence and strong frictions, which mainly dampens the response to shocks but does not generate meaningful inertia in beliefs. This highlights that persistence and information frictions must be jointly identified when estimating macroeconomic models.

Endogenous variables. We now study the propagation of key structural shocks to macroeconomic variables. We focus on investment-specific technology and price mark-up (cost-push) shocks because they account for most of the variation in output and inflation, respectively,

and generate markedly different higher-order belief dynamics. Figures with impulse responses to the remaining shocks are available in Appendix J.

Figures 5-6 report posterior mean IRFs from the FI model (dashed blue line) and the DI model (solid black line), with respective 90% credible intervals shown as shaded bands. We also plot IRFs from the FI model evaluated at the DI posterior mode (dot-dashed red). The difference between the black and red lines isolates the role of information frictions. The remaining gap between the red and blue lines reflects parameter re-estimation.

Figure 5: Impulse responses to an investment-specific technological shock



Note: Posterior-mean IRFs with 90% credible intervals to a one-standard-deviation shock in each model. Revision panels use growth rates for real variables and levels for nominal variables; panels for macroeconomic variables report level responses.

Figure 5 shows impulse responses to a one-standard-deviation investment shock. Investment, output, and hours expand in a hump-shaped manner, while consumption rises more gradually. The inflation response is insignificant on impact and then turns negative as the expansion reverses. The nominal interest rate initially rises with the expansion in activity and subsequently declines as inflation falls and the boom fades.

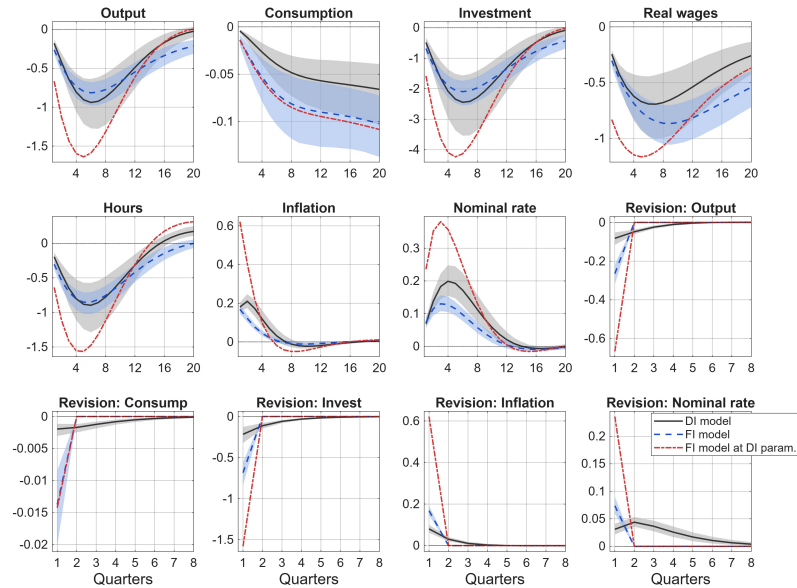
Information frictions dampen the effects of investment-specific shocks and delay their transmission. Holding parameters at the DI posterior, the FI economy (dot-dashed red lines) responds much more strongly and peaks one quarter earlier. Re-estimating the FI model (dashed blue lines) scales down the investment shock, so that the impact response of investment is similar to that in the DI model. The re-estimated FI model nonetheless

generates lower peaks for real variables, a stronger inflation response on impact, and a slightly higher response of the nominal rate.

DI and FI models differ more substantially in expectation dynamics. In the revision panels, the FI model exhibits the textbook full-information pattern: the revision for each variable jumps to match the variable's impact response, and immediately returns to zero afterward. This is one of the key reasons why full-information models cannot adequately fit the expectations data.

Under DI, revision responses are attenuated on impact, unwinding within a quarter for output and investment but closer to a year for consumption and the nominal rate. This reflects strong information frictions combined with the shock's low persistence, implying a lack of sustained inertia in beliefs, as discussed in Figure 4.

Figure 6: Impulse responses to a price mark-up shock



Note: Posterior-mean IRFs with 90% credible intervals to a one-standard-deviation shock in each model. Revision panels use growth rates for real variables and levels for nominal variables; panels for macroeconomic variables report level responses.

For price mark-up (cost-push) shocks (Figure 6), the contrast between the two models is concentrated in the nominal block. Evaluated at the DI posterior, the FI economy (dot-dashed red lines) displays substantially larger declines in output, investment, hours, and real wages, reaches its trough one quarter earlier, and delivers an inflation response more than three times as large on impact. Information frictions therefore mute and delay the transmission of cost-push shocks.

Re-estimation closes most of this gap for real activity (dashed blue versus solid black lines), in part because the re-estimated FI model features a smaller but more persistent mark-up shock (Table 1). On impact, DI and the re-estimated FI model now generate a similar inflation response. Under DI, however, inflation follows hump-shaped dynamics, whereas in the FI model it peaks immediately and declines monotonically from the outset. After a year, the DI response is more than twice as large. As a result, the DI policy rate responds more strongly and peaks one quarter later than in the FI model.

Consistent with Figure 4, the combination of shock persistence and strong information frictions generates attenuated and persistent forecast revisions: under DI the impact revision in output is less than a third of its FI counterpart and takes more than a year to fade, whereas the FI revision returns to zero immediately. Revisions to the nominal interest rate are the most persistent, fading only after roughly two years. This contrasts with the short-lived responses of expectation revisions to investment shocks, consistent with the weaker estimated persistence of those shocks.

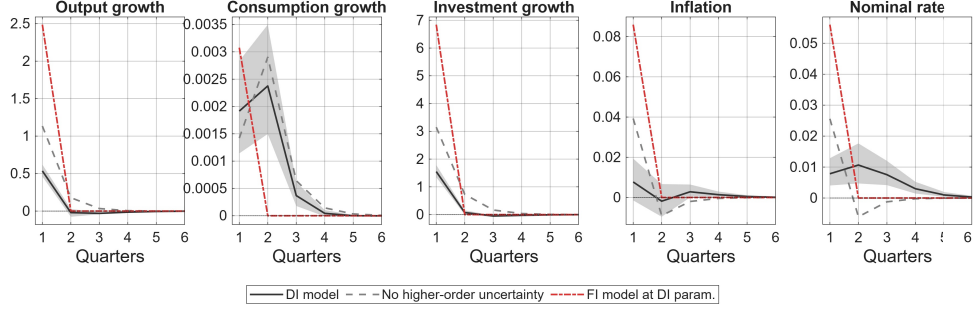
Decomposing the role of information frictions. Incomplete information in our model operates through two channels: gradual learning about exogenous shocks (first-order uncertainty) and uncertainty about other agents' expectations (higher-order uncertainty). We now investigate how each channel shapes the propagation of investment and cost-push shocks.

To disentangle the importance of first-order and higher-order uncertainty, we add one counterfactual IRF (dashed gray) that shuts down higher-order uncertainty by imposing that all orders of beliefs follow the first-order dynamics. The distance between the solid black and dashed gray lines therefore measures the contribution of higher-order uncertainty, and the distance between the dashed gray and dot-dashed red lines (the FI model evaluated at the DI posterior, as above) that of first-order learning.

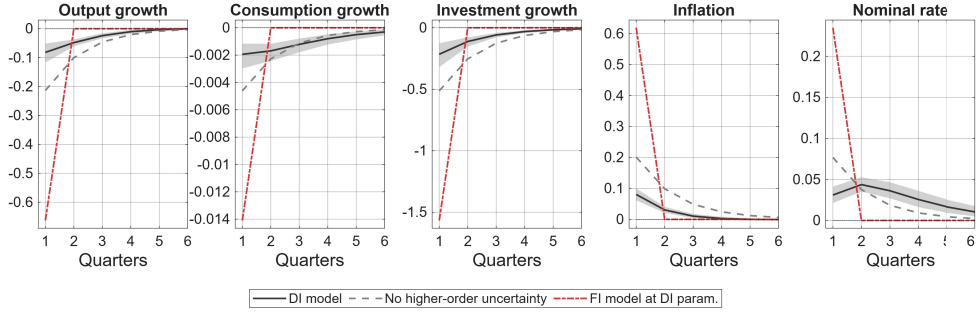
Figure 7 shows the counterfactual exercise for the investment and price mark-up shocks for revision variables. Higher-order uncertainty is quantitatively important for forecast revisions. Under full information, a revision jumps to the variable's impact response and returns to zero immediately. Imperfect learning alone already attenuates that jump substantially, but about a third of total attenuation comes from higher-order uncertainty. Moreover, the revision of the interest-rate forecast is hump-shaped for both shocks, peaking one quarter after impact and fading only gradually. Once higher-order uncertainty is switched off, the same revision peaks on impact and decays monotonically.

How long that inertia lasts mirrors the persistence of the underlying shock. For the weakly persistent investment shock, revisions to real variables are essentially complete after one quarter, and higher-order uncertainty even shortens them: the shock fades before beliefs

Figure 7: Decomposition of impulse responses for revision variables



(a) Investment-specific technology shock



(b) Cost-push shock

about beliefs can build inertia. For the persistent cost-push shock, revisions decay over roughly six quarters.

By contrast, higher-order uncertainty contributes little to the transmission of these shocks to macroeconomic variables. Figure J6 in Appendix J reports the same decomposition for those variables: the DI and no-higher-order-uncertainty lines are nearly indistinguishable. The attenuation and delay documented in Figures 5 and 6 are almost entirely the product of first-order learning.

This limited role reflects the structure of the model. Most forward-looking terms are integrals of individual expectations about agents' own future choices, which do not raise the order of beliefs (Section 2.1). Higher-order expectations enter through a small set of average-expectation terms, and only two of them are self-referential: inflation depends on its own average expectation in the NKPC, and the real wage on its own in the wage NKPC. The coefficients on these terms, $\psi_p \equiv (1 - \xi_p)(1 - \iota_p \beta \xi_p)$ and $\psi_w \equiv (1 - \xi_w)(1 - \beta \xi_w) \frac{\chi \theta_w}{1 + \chi \theta_w}$, where θ_w is the elasticity of substitution across labor varieties (equations 28 and I.12), govern how fast the dependence on higher-order beliefs decays with the order, and both are small at our estimates ($\psi_p = 0.16$, $\psi_w = 0.03$). As a result, second-order beliefs account for over

90 percent of the total higher-order effect, and orders beyond three are negligible. Higher-order beliefs therefore mainly shape the dynamics of forecast revisions, the observables that identify the information frictions, and contribute only modestly to the transmission of shocks to macroeconomic aggregates.

5.4 Information frictions and standard frictions: substitutes or complements?

A large literature argues that informational frictions can serve as an alternative to “standard” frictions, such as habit formation and investment adjustment costs, for explaining the persistence observed in aggregate macroeconomic data. Our quantitative framework provides a natural environment to assess the substitutability hypothesis, as it nests both types of frictions and estimates them jointly under flexible priors.

In this section, we evaluate the relative importance of each friction by re-estimating the model under alternative restricted specifications that remove selected sources of inertia, and then comparing log marginal likelihoods to the baseline. In this experiment, we replace IAC with standard capital adjustment costs (CAC) in the tradition of [Chari et al. \(2002\)](#), so that the model features a friction on capital accumulation based on the investment-to-capital ratio rather than the investment-growth formulation of [Christiano et al. \(2005\)](#) and SW.³⁰

SW originally conducted this exercise using a full-information model estimated on macroeconomic data only, finding that habit formation and investment adjustment costs are key, whereas price and wage indexation play a comparatively minor role. We revisit it for the model with information frictions estimated on the complete dataset.³¹ Table 2 reports results.

Most standard inertial frictions remain dispensable conditional on information frictions: shutting down price indexation and replacing IAC with capital adjustment costs each *improves* fit, while shutting down wage indexation only marginally worsens it (by about 8 log points). The key exception is habit formation, which becomes more important once expectations data are included. Removing habits alone lowers the log marginal likelihood by roughly 90 points, and eliminating all standard frictions jointly lowers it by about 83 points – a deterioration driven entirely by habits.

Why are habits so important when expectations data are included? Inspecting the parameter estimates provides some insight. Shutting down habits moves the preference-shock block along three margins. First, the preference shock becomes far more persistent, with ρ_c

³⁰See Appendix E for the exact specification.

³¹Recall that our sample and investment definitions differ from those in SW.

Table 2: Marginal Likelihoods shutting down key frictions

Model	Marginal Likelihood	Difference from baseline
Baseline	-1381.55	-
Shutting down:		
All standard frictions*	-1464.28	-82.73
Habits	-1474.80	-93.26
Invest. adjustment costs*	-1374.97	6.57
Price indexation	-1366.29	15.25
Wage indexation	-1389.40	-7.85
Full information for:		
All shocks	-1600.49	-218.94
Preference shock	-1470.95	-89.40
Inv. specific tech. shock	-1456.35	-74.80
Gov. expenditure shock	-1430.97	-49.42
Price markup shock	-1420.73	-39.18
Wage markup shock	-1390.03	-8.48
Monetary policy shock	-1401.39	-19.85
TFP shock	-1399.56	-18.02

Notes: Log-marginal likelihoods are computed using the modified harmonic mean estimator of Geweke (1999).

*When shutting down investment adjustment costs, we substitute them with standard capital adjustment costs. See Appendix E for details.

rising from 0.14 to 0.88. Second, and perhaps surprisingly, information frictions on the same shock become *weaker*: the Kalman gain $\bar{k}_c$ more than doubles, from 0.23 to 0.53. Third, the measurement error on consumption revisions rises by 60 percent (see Appendix E).

These three movements are symptoms of a tension: macroeconomic aggregates are highly persistent, whereas current forecast revisions of their growth rates display little persistence and are only weakly related to contemporaneous forecast errors (as suggested by the empirical Kalman gains in Figures 1 and 2). Since preference shocks are the primary driver of consumption fluctuations, the baseline model reconciles these two facts through a clean separation of roles: strong habit formation supplies the intrinsic propagation that makes consumption persistent, while the low persistence of preference shocks, combined with strong information frictions, delivers the weak persistence of consumption growth revisions.

Without habits, the main remaining source of consumption persistence is the shock itself, so ρ_c must rise substantially. But the persistence of average expectations is governed by $(1 - \bar{k}_c)\rho_c$ (equation 26), so the model must attenuate the information friction to preserve the weak persistence of consumption growth revisions. The offset is only partial, raising the persistence from 0.11 to 0.41. Forecast revisions remain far more inertial than in the data,

and the residual mismatch is absorbed by the larger measurement error on consumption revisions. Habit formation therefore provides the intrinsic propagation needed to reconcile the observed persistent consumption dynamics with weakly persistent consumption growth revisions.

By contrast, investment dynamics illustrate a viable form of substitution. Replacing IAC with CAC in the full dataset improves fit marginally, by about 6 log points. The two exercises are structurally identical. First, investment is persistent, and investment growth revisions are weakly persistent. Second, capital adjustment costs impose no penalty on changing the rate of investment, so replacing IAC removes the intrinsic propagation that generates hump-shaped investment, just as removing habits removes it from consumption.

However, the estimated parameter configuration in the CAC model generates a balanced combination of capital adjustment costs, shock persistence, and information frictions that better accommodates the dynamics of investment and of its growth revisions. The model selects sizable capital adjustment costs³² together with, relative to the baseline, a more persistent investment shock (ρ_i rises from 0.42 to 0.80) and weaker information frictions ($\bar{k}_i$ rises from 0.46 to 0.59).³³

The role of information frictions. We also assess the contribution of incomplete information to fitting the complete dataset. Imposing full information for all shocks lowers the log marginal likelihood by about 219 log points, substantially more than the loss from shutting down standard frictions. This indicates that information frictions are central for matching expectations data in our environment.

Across shocks, information frictions on preference and investment-specific shocks are the two most important single components: imposing full information for either one alone lowers the log marginal likelihood by about 89 and 75 log points, respectively (together accounting for the bulk of the full-information loss). Information frictions on government expenditure and cost-push shocks matter as well (drops of about 49 and 39 log points, respectively). Frictions associated with the remaining shocks play a comparatively minor role: imposing full information for the monetary policy, TFP, or wage mark-up shock lowers the log marginal likelihood by about 20, 18, and 8 log points, respectively.

³²The posterior mode of the capital adjustment cost parameter is $s_k'' = 20.8$. Standard calibrations pin this parameter down from steady-state relationships that are common to full- and imperfect-information models. [Bernanke et al. \(1999\)](#) calibrate s_k'' by setting the steady-state elasticity of marginal q with respect to the investment rate to 0.25, and report values in the range $[0, 0.5]$, while noting the lack of consensus in the literature. In our CAC model, the implied elasticity is 0.61, above this range. One interpretation is that informational frictions dampen the response of q , requiring stronger CAC to match observed investment dynamics.

³³The persistence of average expectations about the investment shock, $(1 - \bar{k}_i)\rho_i$, rises only from 0.23 to 0.33, a far smaller change than shutting down habits imposes on its consumption counterpart.

Taking stock. Taken together, the results provide an important qualification to the substitutability view. Standard inertial mechanisms are not essential once incomplete information is allowed: shutting down price indexation or replacing investment adjustment costs with capital adjustment costs each slightly improves fit relative to the baseline, while removing wage indexation lowers it only marginally.

The key exception is habit formation. The reason is that expectations data jointly identify shock persistence and information frictions through the dynamics of forecast revisions. For investment, capital adjustment costs provide sufficient intrinsic propagation to accommodate moderate persistence and moderate information frictions consistent with the revision evidence. For consumption, no such alternative exists: without habits, matching consumption persistence requires highly persistent preference shocks, which generates overly persistent revisions that the expectations data reject.

6 Conclusion

We develop a solution method for dispersed-information DSGE models that accommodates endogenous state variables, and use it to estimate a medium-scale model with Bayesian methods using both macroeconomic and expectations data.

When disciplined by expectations data, the estimation identifies strong information frictions across all structural shocks, consistent with survey-based measures of information rigidity from the SPF. These frictions dampen general equilibrium propagation by slowing the updating of agents' beliefs, making the partial equilibrium effects of each shock more prominent. How this dampening manifests depends on shock persistence: persistent shocks subject to strong frictions generate hump-shaped dynamics in average expectations, which are crucial for the hump-shaped inflation response to cost-push shocks. Less persistent shocks are mainly attenuated on impact.

The interaction between shock persistence and incomplete information also determines which standard inertial mechanisms remain essential. Information frictions can replace most of them: shutting down price indexation or replacing investment adjustment costs with capital adjustment costs each slightly improves fit, while shutting down wage indexation worsens it only marginally. Without habits, matching consumption persistence requires highly persistent preference shocks, which generates overly persistent forecast revisions that the expectations data reject.

Natural extensions of our work include embedding richer informational structures with public signals, and studying the business cycle consequences of belief fluctuations driven by noise shocks ([Lorenzoni, 2010](#); [Blanchard et al., 2013](#)) or confidence shocks ([Angeletos and](#)

La'O, 2013; Angeletos et al., 2018). In conclusion, we see an open avenue for exploring the quantitative potential of macroeconomic models with information frictions.

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Online Appendix for “Expectations and Frictions: Lessons from a Quantitative Dispersed Information Model”

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A Log-linearized model

The log-linearization of the model's optimality conditions leads to the following system of equations. For the derivation of these equations from first-order conditions, including the detrending procedure, see Sections G-I.

$$c_t = \frac{\varphi/\gamma}{1 + \varphi/\gamma} c_{t-1} + \frac{1}{1 + \varphi/\gamma} \int_0^1 E_{h,t}[c_{h,t+1}] dh - \frac{1 - \varphi/\gamma}{1 + \varphi/\gamma} \bar{E}_t[r_t - \pi_{t+1} + (x_{t+1}^c - x_t^c)] \quad (\text{A.1})$$

$$i_t = \left(\frac{1}{1 + \beta} \right) i_{t-1} + \left(\frac{\beta}{1 + \beta} \right) \int_0^1 E_{h,t}[i_{h,t+1}] dh + \left(\frac{1}{s''(1 + \beta)\gamma^2} \right) (q_t + \bar{E}_t[x_t^i]) \quad (\text{A.2})$$

$$q_t = \left(\frac{\beta(1 - \delta)}{\gamma} \right) \int_0^1 E_{h,t}[q_{h,t+1}] dh + \left(1 - \frac{\beta(1 - \delta)}{\gamma} \right) \bar{E}_t[r_{t+1}^k] - \bar{E}_t[r_t - \pi_{t+1}] \quad (\text{A.3})$$

$$k_t = \left(\frac{1 - \delta}{\gamma} \right) k_{t-1} + \left(1 - \frac{1 - \delta}{\gamma} \right) (i_t + x_t^i) \quad (\text{A.4})$$

$$u_t = (\bar{R}^k/a'') \bar{E}_t[r_t^k] \quad (\text{A.5})$$

$$k_t^u = k_{t-1} + u_t \quad (\text{A.6})$$

$$mrs_t = \left(\frac{1}{1 - \varphi/\gamma} \right) (c_t - \varphi/\gamma c_{t-1}) + \chi l_t \quad (\text{A.7})$$

$$y_t = \left(1 + \frac{\Phi_p}{Y} \right) (\alpha k_t^u + (1 - \alpha)l_t + x_t^a) \quad (\text{A.8})$$

$$k_t^u - l_t = w_t - r_t^k \quad (\text{A.9})$$

$$mc_t = (1 - \alpha)w_t + \alpha r_t^k - x_t^a \quad (\text{A.10})$$

$$y_t = \frac{1}{1 - g_y} \left(\frac{\bar{C}}{\bar{Y}} c_t + \frac{\bar{I}}{\bar{Y}} i_t + \frac{\bar{R}^k \bar{K}}{\bar{Y}} u_t + x_t^g \right) \quad (\text{A.11})$$

$$r_t = \phi_r r_{t-1} + (1 - \phi_r) (\phi_\pi \pi_t + \phi_y y_t) + x_t^r \quad (\text{A.12})$$

$$\begin{aligned} \pi_t = & \xi_p \iota_p \pi_{t-1} + \kappa_p \xi_p \bar{E}_t[mc_t + x_t^p] + (1 - \xi_p)(1 - \iota_p \beta \xi_p) \bar{E}_t[\pi_t] \\ & + (1 - \xi_p) \beta \xi_p \int_0^1 E_{i,t}[p_{i,t+1}^*] di \end{aligned} \quad (\text{A.13})$$

$$w_t + \pi_t = \xi_w (\iota_w \pi_{t-1} + w_{t-1}) + (1 - \xi_w) w_t^* \quad (\text{A.14})$$

$$w_t^* = \frac{(1 - \beta \xi_w)}{1 + \chi \theta_w} \bar{E}_t[mrs_t + \chi \theta_w w_t + x_t^w] + (1 - \iota_w \beta \xi_w) \bar{E}_t[\pi_t] + \beta \xi_w \int_0^1 E_{h,t}[w_{h,t+1}^*] dh \quad (\text{A.15})$$

where $\kappa_p = \frac{(1 - \xi_p)(1 - \beta \xi_p)}{\xi_p}$, $\kappa_w = \frac{(1 - \xi_w)(1 - \beta \xi_w)}{\xi_w(1 + \chi \theta_w)}$ and $\theta_w = \frac{1 + \bar{\mu}^w}{\bar{\mu}^w}$. $\bar{E}_t[\cdot] \equiv \int_0^1 E_{i,t}[\cdot] di$ denotes average expectation, where $E_{i,t}[\cdot] \equiv E[\cdot | \mathcal{I}_{it}]$ is agent i 's expectation based on her information

set at period t , $\mathcal{I}_{it}$.

All equations are standard in macroeconomic models. Equation (A.1) is the Euler equation for consumption. Equation (A.2) is the equilibrium condition for investment, and Equation (A.3) determines the marginal real value of a unit of capital q_t . Equation (A.4) is the law of motion of capital. Equation (A.5) is the optimal condition for capital utilization, and Equation (A.6) comes from the definition of utilized capital. Equations (A.7) and (A.8) are the marginal rate of substitution between consumption and labor and the production function, respectively. Equations (A.9) and (A.10) come from firms' cost minimization. Equation (A.11) derives from the aggregate resource constraint, while Equation (A.12) is the Taylor rule. Finally, Equation (A.13) is the New Keynesian Phillips curve for prices, Equation (A.14) describes wage dynamics, and Equation (A.15) is the optimal wage setting condition.

B Signal extraction and steady-state Kalman gain

In section 4 of the main paper, we estimate the steady-state Kalman gain for each shock directly. While the Kalman gain is an endogenous object, our information structure implies that it has a one-to-one relationship to the private signal variance, conditional on the shock persistence and variance. In the following, we show that this is the case.

Recall that all shocks have the same AR(1) structure given by

$$x_t^l = \rho_l x_{t-1}^l + \varepsilon_t^l, \quad \varepsilon_t^l \sim \mathcal{N}(0, \sigma_l^2). \quad (\text{B.1})$$

where $l \in \{a, c, i, g, p, w, r\}$.

Agents use information from their signals, restated here for convenience:

$$s_{j,t}^l = x_t^l + v_{j,t}^l, \quad v_{j,t}^l \sim \mathcal{N}(0, \tau_l^2). \quad (\text{B.2})$$

Agents must form expectations about the whole hierarchy of expectations of all shocks and the solution method takes this into account. Here we simply show how our measure of information friction – the steady-state Kalman gain from the first-order expectation of individual j – can be found as a function of deep parameters.

Since shocks and their respective signals are independent, their signal extraction can be written separately. The rational expectation of each shock l can be computed by the Kalman filter using (B.1) as state equation and (B.2) as measurement equation.

The update equation of the Kalman filter is given by

$$E_{j,t}[x_t^l] = E_{j,t-1}[x_t^l] + \bar{k}_l [s_{j,t}^l - E_{j,t-1}[s_{j,t}^l]] \quad (\text{B.3})$$

where $\bar{k}_l$ is the steady-state Kalman gain. The steady-state Kalman gain for each shock l can be computed by the standard Kalman filter equations applied to the state equation (B.1) and measurement equation (B.2) and evaluated at the steady-state. This implies the following expressions:

$$\bar{k}_l = \frac{\bar{p}_l}{\bar{p}_l + \tau_l^2} \quad (\text{B.4})$$

$$\bar{p}_l = \rho_l^2(1 - \bar{k}_l)\bar{p}_l + \sigma_l^2 \quad (\text{B.5})$$

where $\bar{k}_l$ and $\bar{p}_l$ are the Kalman gain and the mean squared error (MSE) of the one-period ahead prediction error both at steady-state.

Note that the Kalman gain can be expressed alternatively in terms of the MSE relative to the shock variance such that

$$\begin{aligned} \bar{k}_l &= \frac{\tilde{p}_l}{\tilde{p}_l + r_l^2} \\ \tilde{p}_l &= \rho_l^2(1 - \bar{k}_l)\tilde{p}_l + 1 \end{aligned}$$

where $\tilde{p}_l = \bar{p}_l/\sigma_l^2$ is the relative MSE and $r_l = \tau_l/\sigma_l$ the noise-to-signal ratio. The first equation simply divides the denominator and numerator of equation (B.4) by σ_l^2 while the second equation can be found by dividing equation (B.5) by σ_l^2 on both sides.

Both equations lead to the quadratic Riccati equation in terms of the noise-to-signal ratio:

$$\tilde{p}_l^2 - (1 - (1 - \rho_l^2)r_l^2)\tilde{p}_l - r_l^2 = 0 \quad (\text{B.6})$$

This equation also has a unique positive solution, leading to $\tilde{p}_l = \tilde{p}_l(\rho_l, r_l)$.

This implies that the steady-state Kalman filter depends only on the persistence of the shock and the noise-to-signal ratio $\bar{k}_l = \bar{k}_l(\rho_l, r_l) \in (0, 1)$.

C Data

This appendix describes the construction of the observed macroeconomic variables and forecast revisions used in the estimation. Data on macroeconomic aggregates are collected from

the Federal Reserve Database (FRED) and expectations data come from the Survey of Professional Forecasters (SPF). The time series collected to construct the database are displayed in Table C1. The data starts in 1981Q4 due to the expectations data collection and ends in 2007Q4, before the Great Recession, to avoid issues with lower zero bound of the interest rate.

Table C1: Database

Series	Description	Measure	Source
GDPC1	Real Gross Domestic Product	Billions of Chained 2012 Dollars	BEA
PCECC96	Real Personal Consumption Expenditures	Billions of Chained 2012 Dollars	BEA
GPDI1	Real Gross Private Domestic Investment	Billions of Chained 2012 Dollars	BEA
GDPDEF	Gross Domestic Product: Implicit Price Deflator	Index 2012=100	BEA
DTB3	3-Month Treasury Bill Secondary Market Rate	Percentage points	FED Board
PRS85006063	Nonfarm Business Sector: Compensation	Index 2012=100	BEA
HOANBS	Nonfarm Business Sector: Hours of All Persons	Index 2012=100	BLS
CNP16OV	Civilian Noninstitutional Population	Thousands of People	BLS
RGDP	Average forecast for the Real Gross Domestic Product	Chained Dollars (base year varies)	SPF
RCONSUM	Average forecast for the Real Personal Consumption Expenditures	Chained Dollars (base year varies)	SPF
RNRESINV	Average forecast for the Real Nonresidential Fixed Investment	Chained Dollars (base year varies)	SPF
RRESINV	Average forecast for the Real Residential Fixed Investment	Chained Dollars (base year varies)	SPF
RCBI	Average forecast for the Real change in Private Inventories	Chained Dollars (base year varies)	SPF
PGDP	Average forecast for the GDP Price Index	Index (base year varies)	SPF
TBILL	Average forecast for the annualized three-month Treasury bill rate	Percentage points	SPF

Note: BEA, BLS, Fed Board and SPF stand for, respectively: U.S. Bureau of Economic Analysis, U.S. Bureau of Labor Statistics, Board of Governors of the Federal Reserve and Survey of Professional Forecasters.

All variables from the Survey of Professional Forecasters (SPF) are suffixed with a number indicating the forecast horizon. We use horizons 1 through 3, which correspond to the previous quarter, the current quarter, and one-quarter ahead, respectively. For example, RGDP1 is the nowcast of real GDP for the quarter prior to the survey’s release, RGDP2 is the nowcast for the current quarter, and RGDP3 is the one-quarter ahead forecast.³⁴

Real output (GDPC1), consumption (PCE) and investment (GDPIC1) are calculated by dividing them by the Hodrick-Prescott (HP) filtered trend of the population index (CNP16OV), denoted by HP(CNP16OV). This method, as discussed by [Edge and Gürkaynak \(2010\)](#), avoids incorporating fluctuations caused by measurement changes from the Census Bureau.

Real wage is constructed by dividing the compensation in the nonfarm business sector (PRS85006063) by the GDP deflator (GDPDEF). Per capita total hours is computed by dividing hours of all persons in the nonfarm business sector (HOANBS) by the smoothed population index. Inflation is defined as the annualized quarterly growth rate of GDP deflator (GDPDEF). The nominal interest rate is measured by 3-month Treasury bill rate to be

³⁴Although the Bureau of Economic Analysis’s (BEA) advance estimates for the previous quarter are available, forecasters can still “predict” this quarter’s data by forecasting revisions to the official data. According to the survey, most forecasters use the BEA’s advanced estimate.

consistent with the SPF expectations data, as it does not survey forecasts for the federal funds rate.

Therefore, the macroeconomic data in levels is constructed according to:

$$\begin{aligned} Y_t^{obs} &= GDPC1_t / HP(CNP16OV_t) \\ C_t^{obs} &= PCE_t / HP(CNP16OV_t) \\ I_t^{obs} &= GPDIC1_t / HP(CNP16OV_t) \\ W_t^{obs} &= PRS85006063_t / GDPDEF_t \\ L_t^{obs} &= HOANBS_t / HP(CNP16OV_t), \end{aligned}$$

The macroeconomic data used in the estimation is given by:

$$\begin{aligned} dy_t^{obs} &= 100 \log(Y_t^{obs} / Y_{t-1}^{obs}) \\ dc_t^{obs} &= 100 \log(C_t^{obs} / C_{t-1}^{obs}) \\ di_t^{obs} &= 100 \log(I_t^{obs} / I_{t-1}^{obs}) \\ dw_t^{obs} &= 100 \log(W_t^{obs} / W_{t-1}^{obs}) \\ l_t^{obs} &= 100 \left(\log L_t^{obs} - \overline{\log L^{obs}} \right) \\ \pi_t^{obs} &= 400 \log(GDPDEF_t / GDPDEF_{t-1}) \\ r_t^{obs} &= DTB3_t, \end{aligned}$$

where $\bar{L}^{obs}$ denotes the sample mean of total hours per capita, L_t^{obs} .

Now we turn to the expectations data. Investment expectations in the SPF are broken down into three components: nonresidential fixed investment (RNRESIN), residential fixed investment (RRESINV), and change in private inventories (RCBI). Thus, we construct forecasts for Real Gross Private Domestic Investment, $RINVEST_t = RNRESIN_t + RRESINV_t + RCBI_t$, to align with the GPDIC1 index.

It is important to note that the SPF's expectations data is in chained dollars with varying base years, while the actual data is chained to 2012 dollars. Additionally, because the SPF does not survey expectations for the population index, we construct per capita forecast variables using the same index as the real variables. Thus, our measure of expectation revisions on per capita variables is not affected by population growth.

Thus, the expectations data in levels and first differences are given by:

$$\begin{aligned}
\bar{E}_t^{obs}[Y_{t+1}] &= RGDP3_t/HP(CNP16OV_{t+1}) \\
\bar{E}_t^{obs}[Y_t] &= RGDP2_t/HP(CNP16OV_t) \\
\bar{E}_t^{obs}[Y_{t-1}] &= RGDP1_t/HP(CNP16OV_{t-1}) \\
\bar{E}_t^{obs}[C_{t+1}] &= RCONSUM3_t/HP(CNP16OV_{t+1}) \\
\bar{E}_t^{obs}[C_t] &= RCONSUM2_t/HP(CNP16OV_t) \\
\bar{E}_t^{obs}[C_{t-1}] &= RCONSUM1_t/HP(CNP16OV_{t-1}) \\
\bar{E}_t^{obs}[I_{t+1}] &= RINVEST3_t/HP(CNP16OV_{t+1}) \\
\bar{E}_t^{obs}[I_t] &= RINVEST2_t/HP(CNP16OV_t) \\
\bar{E}_t^{obs}[I_{t-1}] &= RINVEST1_t/HP(CNP16OV_{t-1}) \\
\bar{E}_t^{obs}[dy_{t+1}] &= 100 \log \left(\bar{E}_t^{obs}[Y_{t+1}] / \bar{E}_t^{obs}[Y_t] \right) \\
\bar{E}_t^{obs}[dc_{t+1}] &= 100 \log \left(\bar{E}_t^{obs}[C_{t+1}] / \bar{E}_t^{obs}[C_t] \right) \\
\bar{E}_t^{obs}[di_{t+1}] &= 100 \log \left(\bar{E}_t^{obs}[I_{t+1}] / \bar{E}_t^{obs}[I_t] \right) \\
\bar{E}_t^{obs}[dy_t] &= 100 \log \left(\bar{E}_t^{obs}[Y_t] / \bar{E}_t^{obs}[Y_{t-1}] \right) \\
\bar{E}_t^{obs}[dc_t] &= 100 \log \left(\bar{E}_t^{obs}[C_t] / \bar{E}_t^{obs}[C_{t-1}] \right) \\
\bar{E}_t^{obs}[di_t] &= 100 \log \left(\bar{E}_t^{obs}[I_t] / \bar{E}_t^{obs}[I_{t-1}] \right) \\
\bar{E}_t^{obs}[\pi_{t+1}] &= 400 \log(PGDP3_t/PGDP2_t) \\
\bar{E}_t^{obs}[\pi_t] &= 400 \log(PGDP2_t/PGDP1_t) \\
\bar{E}_t^{obs}[r_{t+1}] &= TBILL3_t \\
\bar{E}_t^{obs}[r_t] &= TBILL2_t
\end{aligned}$$

where $\bar{E}_s^{obs}[X_t]$ denotes data on the average expectation about X on horizon t based on information of period s . Growth rates are expressed as percentages, while inflation and the nominal interest rate are in annualized percentage points.

Our primary expectation series is the average current forecast revision, defined as the difference between the nowcast of a variable for the current quarter and the previous quarter's forecast for that same variable. Thus, the expectations data used in the estimation is given

by:

$$\begin{aligned}
\overline{Rev}_{t|t-1}^{obs}[dy_t] &= \bar{E}_t^{obs}[dy_t] - \bar{E}_{t-1}^{obs}[dy_t] \\
\overline{Rev}_{t|t-1}^{obs}[dc_t] &= \bar{E}_t^{obs}[dc_t] - \bar{E}_{t-1}^{obs}[dc_t] \\
\overline{Rev}_{t|t-1}^{obs}[di_t] &= \bar{E}_t^{obs}[di_t] - \bar{E}_{t-1}^{obs}[di_t] \\
\overline{Rev}_{t|t-1}^{obs}[\pi_t] &= \bar{E}_t^{obs}[\pi_t] - \bar{E}_{t-1}^{obs}[\pi_t] \\
\overline{Rev}_{t|t-1}^{obs}[r_t] &= \bar{E}_t^{obs}[r_t] - \bar{E}_{t-1}^{obs}[r_t]
\end{aligned}$$

$\overline{Rev}_{k|s}^{obs}[X_t] = \bar{E}_s^{obs}[X_t] - \bar{E}_{s-1}^{obs}[X_t]$ denotes the revision of the expectation about the same variable X on horizon t based on new information from period k relative to period s .

C.1 Estimation

The log-linearized equilibrium conditions are reported in Section A, while the data construction is detailed in Section C. Below we present the measurement equations linking observables to model variables.

C.1.1 Measurement equations

The following measurement equations link the observed data to the model's variables:

$$\begin{aligned}
dy_t^{obs} &= \bar{\gamma} + y_t - y_{t-1} \\
dc_t^{obs} &= \bar{\gamma} + c_t - c_{t-1} \\
di_t^{obs} &= \bar{\gamma} + i_t - i_{t-1} \\
dw_t^{obs} &= \bar{\gamma} + w_t - w_{t-1} \\
l_t^{obs} &= l_t \\
\pi_t^{obs} &= \bar{\pi} + 4\pi_t \\
r_t^{obs} &= \bar{r} + 4r_t \\
\overline{Rev}_{t|t-1}^{obs}[dy_t] &= \bar{E}_t[\Delta y_t] - \bar{E}_{t-1}[\Delta y_t] + \varepsilon_{dy,t}^{me} \\
\overline{Rev}_{t|t-1}^{obs}[dc_t] &= \bar{E}_t[\Delta c_t] - \bar{E}_{t-1}[\Delta c_t] + \varepsilon_{dc,t}^{me} \\
\overline{Rev}_{t|t-1}^{obs}[di_t] &= \bar{E}_t[\Delta i_t] - \bar{E}_{t-1}[\Delta i_t] + \varepsilon_{di,t}^{me} \\
\overline{Rev}_{t|t-1}^{obs}[\pi_t] &= 4 \left(\bar{E}_t[\pi_t] - \bar{E}_{t-1}[\pi_t] \right) + \varepsilon_{\pi,t}^{me} \\
\overline{Rev}_{t|t-1}^{obs}[r_t] &= 4 \left(\bar{E}_t[r_t] - \bar{E}_{t-1}[r_t] \right) + \varepsilon_{r,t}^{me},
\end{aligned} \tag{C.1}$$

where $\bar{\gamma} \equiv 100(\gamma - 1)$ is the quarterly trend growth rate of productivity. The variables with superscript “obs” correspond to the variables in the database. The notation dx_t^{obs} denotes the quarterly growth rate of the variables $x \in \{y, c, i, w\}$ at period t . The data on hours worked (l_t^{obs}) is the demeaned log level scaled by 100, while inflation (π_t^{obs}) and nominal interest rate (r_t^{obs}) are computed in annual terms. $\bar{\pi} \equiv 400(\bar{\Pi} - 1)$ and $\bar{r} \equiv 400(\bar{R} - 1)$ stand for the steady-state annualized inflation and nominal interest rates, respectively.

The variables $\overline{Rev}_{t|t-1}^{obs}[x_t]$, for $x \in \{dy, dc, di, \pi, r\}$, are the current forecast revision. Measurement errors ε_x^{me} in the observational equations associated with forecast revisions follow an i.i.d. Gaussian distribution with standard deviations denoted by σ_x^{me} , with $x \in \{dy, dc, di, \pi, r\}$.

D Solution method and Algorithm

We first start with two Lemmas. The first writes the first-order expectation of the hierarchy as a linear transformation of the hierarchy.

Lemma 1. *The first-order expectation of the hierarchy of expectations satisfy*

$$E_t^{(1)} \left[x_t^{(0:\bar{k})} \right] = T x_t^{(0:\bar{k})} \quad (\text{D.1})$$

where $T = \begin{bmatrix} 0_{n\bar{k} \times n} & I_{n\bar{k}} \\ 0_{n \times n} & 0_{n \times n\bar{k}} \end{bmatrix}$ is the order transformation matrix.

Proof. By the definition of $x_t^{(0:\bar{k})}$ one can see that $E_t^{(1)} \left[x_t^{(0:\bar{k})} \right] = x_t^{(1:\bar{k}+1)}$. By definition for $\bar{k}$, any order s such that $s > \bar{k}$ does not affect the equilibrium. Then, without loss of generality, one can set $E_t^{(s)}[x_t] = 0$ if $s > \bar{k}$. Therefore, one can rewrite $E_t^{(1)} \left[x_t^{(0:\bar{k})} \right]$ as

$$E_t^{(1)} \left[x_t^{(0:\bar{k})} \right] = \begin{bmatrix} x_t^{(1:\bar{k})} \\ E_t^{(\bar{k}+1)}[x_t] \end{bmatrix} = \begin{bmatrix} x_t^{(1:\bar{k})} \\ 0_{n \times 1} \end{bmatrix} = \begin{bmatrix} 0_{n\bar{k} \times n} & I_{n\bar{k}} \\ 0_{n \times n} & 0_{n \times n\bar{k}} \end{bmatrix} \begin{bmatrix} x_t \\ x_t^{(1:\bar{k})} \end{bmatrix} = T x_t^{(0:\bar{k})}, \quad (\text{D.2})$$

where the first equality uses the definition of $x_t^{(1:\bar{k}+1)}$ and the second equality uses that $E_t^{(\bar{k}+1)}[x_t] = 0$. In the last equality, T is defined accordingly. □

This Lemma builds on Proposition 1 from Online Appendix of [Melosi \(2017\)](#). It explores the truncation of the hierarchy.

The second Lemma does the opposite: writes the hierarchy as a linear transformation of the first-order expectation of the hierarchy.

Lemma 2. $x_t^{(0:\bar{k})}$ can be rewritten as a linear function of x_t and its average expectation such that

$$x_t^{(0:\bar{k})} \equiv e'_x x_t + T' E_t^{(1)} \left[x_t^{(0:\bar{k})} \right],$$

Proof. By definition, $x_t^{(0:\bar{k})}$ can be decomposed as $x_t^{(0:\bar{k})} = \begin{bmatrix} x'_t & (x_t^{(\bar{1}:\bar{k})})' \end{bmatrix}'$. Using this decomposition, the average expectation decomposed as $E_t^{(1)} \left[x_t^{(0:\bar{k})} \right] = \begin{bmatrix} (x_t^{(\bar{1}:\bar{k})})' & E_t^{(\bar{k}+1)} [x_t] \end{bmatrix}'$. Therefore, one can use this two definitions such that

$$x_t^{(0:\bar{k})} \equiv \begin{bmatrix} x_t \\ x_t^{(\bar{1}:\bar{k})} \end{bmatrix} = \begin{bmatrix} I_n \\ 0_{n\bar{k} \times n} \end{bmatrix} x_t + \begin{bmatrix} 0_{n \times n\bar{k}} & 0_{n \times n} \\ I_{n\bar{k}} & 0_{n\bar{k} \times n} \end{bmatrix} \begin{bmatrix} x_t^{(1:\bar{k})} \\ E_t^{(\bar{k}+1)} [x_t] \end{bmatrix} = e'_x x_t + T' E_t^{(1)} \left[x_t^{(0:\bar{k})} \right],$$

where last equality uses the definitions of T in Lemma 1 and e_x . $\square$

Lemmas are taken from [Ribeiro \(2018, chap. 3\)](#).

D.1 Proof of Proposition 1

Individual and average expectations about the hierarchy. The Kalman filter delivers the individual conditional expectation, $E_{i,t}[\cdot] \equiv E[\cdot | \mathcal{I}_t^i]$, where $\mathcal{I}_t^i = \{s_{i,\tau}, \tau \leq t\}$ is the information set of individual i in period t .

The state equation is the hierarchy of expectations that is given by the state equation (34), restated for convenience:

$$x_t^{(0:\bar{k})} = \mathbf{A} x_{t-1}^{(0:\bar{k})} + \mathbf{B} \varepsilon_t, \quad (\text{D.3})$$

and the agent i with the observational equation (31), also restated:

$$s_{i,t} = C x_t + D v_{i,t}.$$

Note that $s_{i,t}$ is a signal about the shocks and not the whole hierarchy of expectations.

Let the selection matrix $e_x \equiv \begin{bmatrix} I_n & 0_{n \times n\bar{k}} \end{bmatrix}$ such that $x_t = e_x x_t^{(0:\bar{k})}$. Then, we can rewrite the signal in terms of the hierarchy as

$$s_{i,t} = C x_t^{(0:\bar{k})} + D v_{i,t}. \quad (\text{D.4})$$

where $C = C_x e_x$.

Each agent i uses the Kalman filter and finds the update equation given by

$$E_{i,t} \left[x_t^{(0:\bar{k})} \right] = E_{i,t-1} \left[x_t^{(0:\bar{k})} \right] + \mathbf{K}_t [s_{i,t} - E_{i,t-1} [s_{i,t}]], \quad (\text{D.5})$$

where $\mathbf{K}_t$ is the Kalman gain given by

$$\mathbf{K}_t = \mathbf{P}_{t/t-1} C' \left[C \mathbf{P}_{t/t-1} C' + D \Sigma_v D' \right]^{-1}. \quad (\text{D.6})$$

As usual, the mean squared error (MSE) of the one-period ahead prediction error is given by

$$\mathbf{P}_{t+1/t} = \mathbf{A} \left[\mathbf{P}_{t/t-1} - \mathbf{K}_t \left[C \mathbf{P}_{t/t-1} \right] \right] \mathbf{A}' + \mathbf{B} \Sigma_\varepsilon \mathbf{B}'. \quad (\text{D.7})$$

For details of this derivation, see for instance [Hamilton \(1995, chap. 13\)](#).

Using the observational equation, (D.4), taking expectations and inserting in (D.5) one can find:

$$E_{i,t} \left[x_t^{(0:\bar{k})} \right] = E_{i,t-1} \left[x_t^{(0:\bar{k})} \right] + \mathbf{K}_t \left[C x_t^{(0:\bar{k})} + D v_{it} - C E_{i,t-1} \left[x_t^{(0:\bar{k})} \right] \right] \quad (\text{D.8})$$

Therefore, one can rewrite the equation above as

$$E_{i,t} \left[x_t^{(0:\bar{k})} \right] = (I_k - \mathbf{K}_t C) E_{i,t-1} \left[x_t^{(0:\bar{k})} \right] + \mathbf{K}_t \left[C x_t^{(0:\bar{k})} + D v_{it} \right] \quad (\text{D.9})$$

where $k = n(\bar{k} + 1)$. Using the fact that $E_{i,t-1} \left[x_t^{(0:\bar{k})} \right] = \mathbf{A} E_{i,t-1} \left[x_{t-1}^{(0:\bar{k})} \right]$ and substituting equation (D.3), one can find:

$$E_{i,t} \left[x_t^{(0:\bar{k})} \right] = (I_k - \mathbf{K}_t C) \mathbf{A} E_{i,t-1} \left[x_{t-1}^{(0:\bar{k})} \right] + \mathbf{K}_t C \mathbf{A} x_{t-1}^{(0:\bar{k})} + \mathbf{K}_t C \mathbf{B} \varepsilon_t + \mathbf{K}_t D v_{it} \quad (\text{D.10})$$

We follow the literature by focusing on the stationary equilibrium. Therefore, the expectation of each individual i in the stationary equilibrium is the one which the MSE is in steady-state, i.e., agents update their forecast based on the steady-state Kalman gain. In other words, the dynamics of expectations depends only on the properties of the process they are forecasting and signals, but does not depend on the period t .

Combining equations (D.6-D.7), one can find the Riccati equation

$$\mathbf{P}_{t+1/t} = \mathbf{A} \left[\mathbf{P}_{t/t-1} - \mathbf{P}_{t/t-1} C' \left[C \mathbf{P}_{t/t-1} C' + D \Sigma_v D' \right]^{-1} C \mathbf{P}_{t/t-1} \right] \mathbf{A}' + \mathbf{B} \Sigma_\varepsilon \mathbf{B}' \quad (\text{D.11})$$

Therefore, one need to iterate this equation to find the steady-state MSE, $\bar{\mathbf{P}}$, and compute its counterpart Kalman gain, $\bar{\mathbf{K}}$. [Nimark \(2017\)](#) shows that if x_t is stationary process, then the expectations hierarchy about this process, $x_t^{(0:\bar{k})}$, is also stationary. Given this and the fact that Σ_ε is positive definite, there exists a steady-state solution such that $\bar{\mathbf{P}} = \mathbf{P}_{t+1|t} = \mathbf{P}_{t|t-1}$ which implies the steady-state Kalman gain $\bar{\mathbf{K}} = \mathbf{K}_t = \mathbf{K}_{t-1}$ (see [Hamilton, 1995, chap.](#)

13).

Expressions (36) in the Proposition 1 are equations (D.6-D.7) using the steady-state Kalman gain, $\bar{\mathbf{K}}$, instead of $\mathbf{K}_t$.

Moreover, the average expectation is easily computed by

$$\bar{E}_t \left[x_t^{(0:\bar{k})} \right] \equiv \int_0^1 E_{i,t} \left[x_t^{(0:\bar{k})} \right] di = \left(I_k - \bar{\mathbf{K}}C \right) \mathbf{A} \bar{E}_{t-1} \left[x_{t-1}^{(0:\bar{k})} \right] + \bar{\mathbf{K}}C \mathbf{A} x_{t-1}^{(0:\bar{k})} + \bar{\mathbf{K}}C \mathbf{B} \varepsilon_t \quad (\text{D.12})$$

Analogously, the individual and average expectation of Proposition 1 are equations (D.10) and (D.12) using the steady-state Kalman gain, $\bar{\mathbf{K}}$, instead of $\mathbf{K}_t$.

Verify guess for $x_t^{(0:\bar{k})}$. In the first part of the proof, we found the average expectation (D.12) for the guessed dynamics of the hierarchy of expectations (D.3).

Now we verify the guessed hierarchy and find the coefficients ($\mathbf{A}, \mathbf{B}$) consistent with the average expectation.

Substituting the average expectation from equation (D.12) into the expression from Lemma 2 one can find that:

$$x_t^{(0:\bar{k})} = e'_x x_t + T' \left[\left(I_k - \bar{\mathbf{K}}C \right) \mathbf{A} E_{t-1}^{(1)} \left[x_{t-1}^{(0:\bar{k})} \right] + \bar{\mathbf{K}}C \mathbf{A} x_{t-1}^{(0:\bar{k})} + \bar{\mathbf{K}}C \mathbf{B} \varepsilon_t \right]$$

Then, using the shocks definition (30) and the fact that $x_t = e_x x_t^{(0:\bar{k})}$ one can rewrite equation above as

$$x_t^{(0:\bar{k})} = e'_x \left(A_1 e_x x_{t-1}^{(0:\bar{k})} + \varepsilon_t \right) + T' \left(I_k - \bar{\mathbf{K}}C \right) \mathbf{A} E_{t-1}^{(1)} \left[x_{t-1}^{(0:\bar{k})} \right] + T' \bar{\mathbf{K}}C \mathbf{A} x_{t-1}^{(0:\bar{k})} + T' \bar{\mathbf{K}}C \mathbf{B} \varepsilon_t$$

Using Lemma 1 at period $t - 1$ into equation above and rearranging:

$$x_t^{(0:\bar{k})} = \left[e'_x A_1 e_x + T' \left(I_k - \bar{\mathbf{K}}C \right) \mathbf{A} T + T' \bar{\mathbf{K}}C \mathbf{A} \right] x_{t-1}^{(0:\bar{k})} + \left[T' \bar{\mathbf{K}}C \mathbf{B} + e'_x \right] \varepsilon_t. \quad (\text{D.13})$$

This expression shows that the expectations hierarchy is a function of its lag and structural shocks, as guessed in equation (D.3). Therefore, the expression above verifies that $x_t^{(0:\bar{k})}$ follows the guessed form and the square brackets terms provide identities for $\mathbf{A}$ and $\mathbf{B}$ in equations (35)

□

D.2 Proof of Proposition 2

This proof builds on techniques developed by [Ribeiro \(2018, chap. 3\)](#). The key difference is that we guess a law of motion for individual endogenous variables instead of guessing the aggregate. This allows recovering expressions for solving for $(\mathbf{Q}_0, \mathbf{Q}_1)$ and then get the solution for $\mathbf{Q}$. We also consider the exogenous information case only.

The guessed law of motion for individual endogenous variables is given [\(32\)](#), restated for convenience:

$$Y_{i,t} = \mathbf{R}Y_{i,t-1} + \mathbf{Q}_0x_t + \mathbf{Q}_1E_{i,t} \left[x_t^{(0:\bar{k})} \right]. \quad (\text{D.14})$$

Aggregating the individual law of motion [\(D.14\)](#):

$$Y_t = \mathbf{R}Y_{t-1} + \mathbf{Q}_0x_t + \mathbf{Q}_1E_t^{(1)} \left[x_t^{(0:\bar{k})} \right] \quad (\text{D.15})$$

$$= \mathbf{R}Y_{t-1} + \mathbf{Q}_0x_t + \mathbf{Q}_1T x_t^{(0:\bar{k})} \quad (\text{D.16})$$

where the last equality uses [Lemma 1](#).

By computing the individual expectation of the endogenous aggregate variables, one can find that

$$\begin{aligned} E_{i,t} [Y_t] &= \mathbf{R}Y_{t-1} + \mathbf{Q}_0E_{i,t} [x_t] + \mathbf{Q}_1TE_{i,t} \left[x_t^{(0:\bar{k})} \right] \\ &= \mathbf{R}Y_{t-1} + (\mathbf{Q}_0e_x + \mathbf{Q}_1T)E_{i,t} \left[x_t^{(0:\bar{k})} \right] \end{aligned} \quad (\text{D.17})$$

$$E_t^{(1)} [Y_t] = \mathbf{R}Y_{t-1} + (\mathbf{Q}_0e_x + \mathbf{Q}_1T)E_t^{(1)} \left[x_t^{(0:\bar{k})} \right]$$

where the first equality uses that Y_{t-1} is known, second equality uses the definition of the selection matrix, e_x , and the third equality aggregates.

Similarly, the individual expectation for endogenous variables in $t+1$:

$$\begin{aligned} E_{i,t} [Y_{t+1}] &= \mathbf{R}E_{i,t} [Y_t] + \mathbf{Q}_0E_{i,t} [x_{t+1}] + \mathbf{Q}_1TE_{i,t} \left[x_{t+1}^{(0:\bar{k})} \right] \\ &= \mathbf{R}E_{i,t} [Y_t] + (\mathbf{Q}_0A_1e_x + \mathbf{Q}_1T\mathbf{A})E_{i,t} \left[x_t^{(0:\bar{k})} \right] \\ &= \mathbf{R}^2Y_{t-1} + [\mathbf{R}(\mathbf{Q}_0e_x + \mathbf{Q}_1T) + (\mathbf{Q}_0A_1e_x + \mathbf{Q}_1T\mathbf{A})] E_{i,t} \left[x_t^{(0:\bar{k})} \right] \\ E_t^{(1)} [Y_{t+1}] &= \mathbf{R}^2Y_{t-1} + [\mathbf{R}(\mathbf{Q}_0e_x + \mathbf{Q}_1T) + (\mathbf{Q}_0A_1e_x + \mathbf{Q}_1T\mathbf{A})] E_t^{(1)} \left[x_t^{(0:\bar{k})} \right] \end{aligned} \quad (\text{D.18})$$

where the second equality uses equations [\(30\)](#) and [\(34\)](#), and the definition of e_x . Third equation uses equation [\(D.17\)](#) and the fourth aggregates.

Finally, taking the individual expectation of the law of motion [\(32\)](#) in period $t+1$ implies

that

$$\begin{aligned} E_{i,t}[Y_{i,t+1}] &= \mathbf{R}Y_{i,t} + \mathbf{Q}_0 E_{i,t}[x_{t+1}] + \mathbf{Q}_1 E_{i,t} \left[E_{i,t+1} \left[x_{t+1}^{(0;\bar{k})} \right] \right] \\ &= \mathbf{R}Y_{i,t} + (\mathbf{Q}_0 A_1 e_x + \mathbf{Q}_1 \mathbf{A}) E_{i,t} \left[x_t^{(0;\bar{k})} \right] \end{aligned}$$

where the second equality uses the law of iterated expectations (which holds for i's expectation but not for the average expectation), equations (30) and (34) as before.

Aggregating the expectation above leads to

$$\begin{aligned} \int_0^1 E_{i,t}[Y_{i,t+1}] di &= \mathbf{R}Y_t + (\mathbf{Q}_0 A_1 e_x + \mathbf{Q}_1 \mathbf{A}) E_t^{(1)}[x_t^{(0;\bar{k})}] \\ &= \mathbf{R} \left[\mathbf{R}Y_{t-1} + \mathbf{Q}_0 x_t + \mathbf{Q}_1 E_t^{(1)} \left[x_t^{(0;\bar{k})} \right] \right] + (\mathbf{Q}_0 A_1 e_x + \mathbf{Q}_1 \mathbf{A}) E_t^{(1)}[x_t^{(0;\bar{k})}] \\ &= \mathbf{R}^2 Y_{t-1} + \mathbf{R} \mathbf{Q}_0 x_t + [\mathbf{R} \mathbf{Q}_1 + (\mathbf{Q}_0 A_1 e_x + \mathbf{Q}_1 \mathbf{A})] E_t^{(1)}[x_t^{(0;\bar{k})}] \end{aligned} \quad (\text{D.19})$$

where the second equality uses equation (D.16). Note that $\int_0^1 E_{i,t}[Y_{i,t+1}] di \neq E_t^{(1)}[Y_{t+1}]$.

Substituting the guessed solution (D.16) and the expectations (D.17-D.19) into the system of equations (29) one can find:

$$\begin{aligned} &F_1 \left[\mathbf{R}^2 Y_{t-1} + \mathbf{R} \mathbf{Q}_0 x_t + [\mathbf{R} \mathbf{Q}_1 + (\mathbf{Q}_0 A_1 e_x + \mathbf{Q}_1 \mathbf{A})] E_t^{(1)}[x_t^{(0;\bar{k})}] \right] + \\ &F_2 \left[\mathbf{R}^2 Y_{t-1} + [\mathbf{R}(\mathbf{Q}_0 e_x + \mathbf{Q}_1 T) + (\mathbf{Q}_0 A_1 e_x + \mathbf{Q}_1 T \mathbf{A})] E_t^{(1)} \left[x_t^{(0;\bar{k})} \right] \right] + \\ &G_1 \left[\mathbf{R}Y_{t-1} + \mathbf{Q}_0 x_t + \mathbf{Q}_1 E_t^{(1)} \left[x_t^{(0;\bar{k})} \right] \right] + G_2 \left[\mathbf{R}Y_{t-1} + (\mathbf{Q}_0 e_x + \mathbf{Q}_1 T) E_t^{(1)} \left[x_t^{(0;\bar{k})} \right] \right] + \\ &HY_{t-1} + [(LA_1 + M_2)] e_x E_t^{(1)} \left[x_t^{(0;\bar{k})} \right] + M_1 x_t = 0_{m \times 1} \end{aligned}$$

which can be rearranged to

$$\begin{aligned} &[F\mathbf{R}^2 + G\mathbf{R} + H] Y_{t-1} + [(F_1 \mathbf{R} + G_1) \mathbf{Q}_0 + M_1] x_t \\ &[F_1 [\mathbf{R} \mathbf{Q}_1 + (\mathbf{Q}_0 A_1 e_x + \mathbf{Q}_1 \mathbf{A})] + F_2 [\mathbf{R}(\mathbf{Q}_0 e_x + \mathbf{Q}_1 T) + (\mathbf{Q}_0 A_1 e_x + \mathbf{Q}_1 T \mathbf{A})] + \\ &G_1 \mathbf{Q}_1 + G_2(\mathbf{Q}_0 e_x + \mathbf{Q}_1 T) + (LA_1 + M_2)e_x] E_t^{(1)} \left[x_t^{(0;\bar{k})} \right] = 0_{m \times 1} \end{aligned}$$

which can be simplified to

$$\begin{aligned} &[F\mathbf{R}^2 + G\mathbf{R} + H] Y_{t-1} + [(F_1 \mathbf{R} + G_1) \mathbf{Q}_0 + M_1] x_t + \\ &[[F_1 \mathbf{R} + G_1] \mathbf{Q}_1 + F_1 \mathbf{Q}_1 \mathbf{A} + (F_2 \mathbf{R} + G_2) \mathbf{Q}_1 T + F_2 \mathbf{Q}_1 T \mathbf{A} \\ &[(F_2 \mathbf{R} + G_2) \mathbf{Q}_0 + (F_1 + F_2) \mathbf{Q}_0 A_1 + (LA_1 + M_2)] e_x] E_t^{(1)} \left[x_t^{(0;\bar{k})} \right] = 0_{m \times 1} \end{aligned}$$

This condition must hold for all realizations of Y_{t-1} , x_t and $E_t^{(1)} \left[x_t^{(0:\bar{k})} \right]$. Therefore, all coefficients between square brackets must be zero, which leads to

$$\begin{aligned} F\mathbf{R}^2 + G\mathbf{R} + H &= 0_{m \times m} \\ [F_1\mathbf{R} + G_1]\mathbf{Q}_0 + M_1 &= 0_{m \times n} \\ [F_1\mathbf{R} + G_1]\mathbf{Q}_1 + F_1\mathbf{Q}_1\mathbf{A} + (F_2\mathbf{R} + G_2)\mathbf{Q}_1T + F_2\mathbf{Q}_1T\mathbf{A} \\ [(F_2\mathbf{R} + G_2)\mathbf{Q}_0 + F\mathbf{Q}_0A_1 + (LA_1 + M_2)]e_x &= 0_{m \times k} \end{aligned}$$

which leads to the same equations from Proposition 2. $\mathbf{R}$ can be solved using Uhlig (2001) method. For a given solution of $\mathbf{R}$, $\mathbf{Q}_0$ and $\mathbf{Q}_1$ can be solved by straightforward vectorization as discussed in the Proposition. □

D.3 Algorithm

Algorithm. Set the initial values $(\mathbf{A}^{(0)}, \mathbf{B}^{(0)})$, a small tolerance $\epsilon > 0$ and set $i = 1$. Then, follow the steps:

1. Given $\mathbf{A} = \mathbf{A}^{(i-1)}$ and $\mathbf{B}^{(i-1)}$, compute $\bar{\mathbf{K}}$ and $\bar{\mathbf{P}}$ using equations (36) using standard solver for Riccati equations. Set $\bar{\mathbf{K}}^{(i)} = \bar{\mathbf{K}}$ and $\bar{\mathbf{P}}^{(i)} = \bar{\mathbf{P}}$.
2. Given $\mathbf{A}^{(i-1)}$ and $\bar{\mathbf{K}}^{(i)}$, compute the right hand side of equations (35) and solve for $\mathbf{B}$ and $\mathbf{A}$ the equations by matrix inversion. Set $\mathbf{B}^{(i)} = \mathbf{B}$, $\mathbf{A}^{(i)} = \mathbf{A}$.
3. If $\max \left\{ \|\mathbf{B}^{(i)} - \mathbf{B}^{(i-1)}\|, \|\mathbf{A}^{(i)} - \mathbf{A}^{(i-1)}\|, \|\bar{\mathbf{P}}^{(i)} - \bar{\mathbf{P}}^{(i-1)}\| \right\} < \epsilon$, stop iterating. Otherwise, set $i = i + 1$ and go back to step 1.

Given the solution for $\mathbf{A}$, one can use standard techniques for solving for $(\mathbf{R}, \mathbf{Q}_0, \mathbf{Q}_1, \mathbf{Q})$ using Proposition 2.

E Posterior estimates when shutting down frictions

Table 5 in Section 5.2 of the main paper shows the marginal likelihood of the models shutting down frictions that generate inertia re-estimated for both datasets. In this appendix, we show the details of the posterior estimates of each of those models. The priors are the same as the baseline for all unaffected parameters, with the exception of the exercise where we shut down investment adjustment costs.

If we completely shut down investment adjustment costs, investment would freely adjust to shocks, which is a well-known issue for explaining the investment data. Thus, when shutting down investment adjustment costs, we add capital adjustment costs. In the following, we briefly show which equations are changed in the model and in the log-linear version. The details of the derivation are in Sections G–I.

CAC model In the baseline model, we use the following law of motion (Equation 10 in the main paper), reproduced here for convenience:

$$K_{h,t} = (1 - \delta)K_{h,t-1} + e^{x_t^i} (1 - S(I_{h,t}/I_{h,t-1})) I_{h,t}, \quad (\text{E.1})$$

where $S(I_t/I_{t-1})$ is the adjustment investment cost function such that $S(\gamma) = S'(\gamma) = 0$, and $S''(\gamma) = s'' > 0$, as in [Christiano et al. \(2005\)](#).

In the model with standard capital adjustment costs in the tradition of [Bernanke et al. \(1999\)](#) and [Chari et al. \(2002\)](#), the law of motion for capital is the following:

$$K_{h,t} = \left(1 - \delta - S_k \left(\frac{I_{h,t}}{K_{h,t-1}} \right)\right) K_{h,t-1} + e^{x_t^i} I_{h,t}. \quad (\text{E.2})$$

Note that $S_k(\cdot)$ is the capital adjustment cost (CAC) function such that $S_k(\gamma - 1 + \delta) = S'_k(\gamma - 1 + \delta) = 0$, and $S''_k(\gamma - 1 + \delta) = s''_k > 0$. s''_k is the parameter governing capital adjustment costs. When $\gamma = 1$ (there is no balanced growth for productivity), the specification is the same as in [Chari et al. \(2002\)](#).

In the presence of capital adjustment costs, the first-order conditions for capital and investment are, respectively:

$$\begin{aligned} E_{h,t}[\Phi_{h,t}] &= \beta E_{h,t} [\Lambda_{h,t+1} (R_{k,t+1} U_{h,t+1} - P_{t+1} a(U_{h,t+1}))] \\ &+ \beta E_{h,t} \left[\Phi_{h,t+1} \left[1 - \delta - S_k \left(\frac{I_{h,t+1}}{K_{h,t}} \right) + S'_k \left(\frac{I_{h,t+1}}{K_{h,t}} \right) \frac{I_{h,t+1}}{K_{h,t}} \right] \right] \end{aligned} \quad (\text{E.3})$$

$$E_{h,t}[\Phi_{h,t} e^{x_t^i}] = E_{h,t} \left[\Phi_{h,t} \left(1 + S'_k \left(\frac{I_{h,t}}{K_{h,t-1}} \right) \right) \right]. \quad (\text{E.4})$$

In this version, the log-linearized investment equation becomes:

$$i_t = k_{t-1} + \left(\frac{1}{s''_k(\gamma - (1 - \delta))} \right) q_t + \bar{E}_t[\hat{x}_t^i] \quad (\text{E.5})$$

and the log-linearized version of equation of the capital pricing equation becomes:

$$q_t = \left(1 - \frac{(1-\delta)\beta}{\gamma}\right) \bar{E}_t[r_{t+1}^k] + \left(\frac{(1-\delta)\beta}{\gamma}\right) \int_0^1 E_{h,t}[q_{h,t+1}] dh - \bar{E}_t[r_t - \pi_{t+1}] \\ + \beta \gamma s_k'' \left(1 - \frac{1-\delta}{\gamma}\right)^2 \left(\int_0^1 E_{h,t}[i_{h,t+1}] dh - \bar{E}_t[k_t]\right) \quad (\text{E.6})$$

where $q_t = \int_0^1 [q_{h,t}] dh$ is the average expected shadow price of capital.

For estimating the CAC model, we need to define a prior distribution for s_k'' . We follow the same approach used to choose the prior for IAC. We assume a Gamma distribution with mean 4, which is the distribution and mean from the IAC model for s'' . This leads to a standard deviation for investment growth rate that is at par with the baseline model. We decide to use a flatter prior due to the lack of medium-scale DSGE models estimated with CAC, leading to a standard deviation of 2.5 (we use 1.5 for the IAC).

Table E1 shows the posterior estimates when shutting down standard frictions for the complete dataset. In the specification without standard frictions, habit formation and price and wage indexation are shut down, and investment adjustment costs are replaced by capital adjustment costs.

Table E1: Posterior distributions when shutting down key standard frictions: Complete dataset

Parameters	Baseline			No standard frictions			No habits			No inv. adj. costs			No price idx.			No wage idx.		
	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
<i>Endogenous propagation parameters</i>																		
χ	1.51	0.92	2.14	0.52	0.32	0.85	0.67	0.44	1.08	0.98	0.59	1.92	1.40	1.03	6.43	0.83	0.41	1.73
φ	1.00	0.99	1.00	-	-	-	-	-	-	1.00	0.98	1.00	1.00	0.99	1.00	0.99	0.97	0.99
a''	0.06	0.06	0.22	0.38	0.28	0.66	0.31	0.20	0.57	0.27	0.17	0.54	0.06	0.06	0.17	0.25	0.15	0.40
s''	4.83	3.62	6.80	-	-	-	2.01	1.42	3.08	-	-	-	5.19	3.84	6.96	3.45	2.47	4.70
s_k''	-	-	-	7.25	6.00	10.05	-	-	-	20.78	19.64	21.89	-	-	-	-	-	-
ϕ_y	0.06	0.04	0.11	0.39	0.30	0.58	0.28	0.21	0.41	0.41	0.32	0.59	0.06	0.03	0.13	0.29	0.22	0.45
ϕ_π	3.02	2.43	3.88	0.50	0.33	0.82	0.91	0.65	1.32	2.28	1.86	2.91	3.13	2.54	4.05	1.41	1.06	1.96
ϕ_r	0.87	0.84	0.90	0.64	0.51	0.72	0.79	0.72	0.85	0.92	0.89	0.94	0.88	0.84	0.91	0.91	0.87	0.93
t_p	0.03	0.01	0.13	-	-	-	0.14	0.06	0.29	0.06	0.02	0.18	-	-	-	0.13	0.06	0.52
t_w	0.76	0.37	0.91	-	-	-	0.17	0.06	0.50	0.29	0.10	0.67	0.68	0.27	0.89	-	-	-
ξ_p	0.84	0.80	0.87	0.95	0.94	0.96	0.94	0.92	0.95	0.94	0.92	0.95	0.83	0.79	0.87	0.95	0.92	0.96
ξ_w	0.81	0.78	0.85	0.80	0.74	0.84	0.78	0.71	0.84	0.89	0.85	0.92	0.81	0.73	0.84	0.87	0.82	0.91
<i>Steady-state parameters</i>																		
π_{ss}	2.07	1.56	2.72	2.43	1.83	3.00	1.94	1.50	2.52	1.74	1.32	2.36	2.35	1.72	2.97	1.96	1.50	2.64
r_{ss}	6.58	5.83	7.62	6.04	5.42	6.65	5.41	4.89	6.07	5.80	5.17	6.59	6.37	5.42	7.43	5.51	4.90	6.13
γ_{ss}	0.36	0.31	0.41	0.36	0.32	0.40	0.49	0.46	0.52	0.46	0.44	0.48	0.36	0.30	0.41	0.48	0.46	0.50
<i>Structural shocks parameters</i>																		
ρ_c	0.14	0.07	0.21	0.79	0.72	0.86	0.88	0.82	0.94	0.14	0.07	0.22	0.13	0.07	0.22	0.15	0.09	0.24
ρ_i	0.42	0.33	0.50	0.82	0.79	0.86	0.36	0.30	0.43	0.80	0.75	0.83	0.40	0.34	0.49	0.41	0.34	0.48
ρ_g	0.97	0.95	0.99	0.96	0.94	0.97	0.96	0.94	0.97	0.98	0.97	0.99	0.98	0.95	0.99	0.98	0.96	0.98
ρ_p	0.74	0.65	0.81	0.52	0.44	0.60	0.41	0.29	0.51	0.55	0.43	0.67	0.80	0.67	0.86	0.40	0.19	0.51
ρ_w	0.17	0.09	0.25	0.27	0.16	0.40	0.32	0.21	0.48	0.27	0.18	0.38	0.17	0.09	0.28	0.35	0.24	0.48
ρ_r	0.44	0.31	0.57	0.92	0.89	0.94	0.77	0.64	0.84	0.43	0.32	0.55	0.42	0.29	0.56	0.42	0.29	0.57
ρ_a	1.00	1.00	1.00	0.99	0.98	0.99	0.99	0.98	0.99	1.00	0.99	1.00	1.00	0.93	1.00	1.00	0.99	1.00
σ_c	1.06	0.87	1.52	1.48	1.29	1.78	1.71	1.48	2.17	1.00	0.80	1.38	1.05	0.83	1.45	1.05	0.84	1.45
σ_i	2.73	2.37	3.26	10.51	9.01	13.32	3.77	3.31	4.56	7.22	6.48	8.35	2.66	2.31	3.22	3.12	2.64	3.70
σ_g	0.55	0.49	0.65	0.61	0.53	0.70	0.65	0.58	0.75	0.59	0.52	0.68	0.60	0.51	0.70	0.63	0.56	0.72
σ_p	0.26	0.20	0.41	1.27	0.80	2.68	0.70	0.48	1.74	0.39	0.26	0.69	0.27	0.18	0.39	0.44	0.32	0.77
σ_w	0.94	0.72	1.76	0.57	0.44	1.19	0.55	0.40	1.44	0.80	0.59	1.78	0.96	0.70	1.74	0.61	0.41	1.31
σ_r	0.13	0.12	0.15	0.16	0.14	0.19	0.11	0.10	0.13	0.12	0.11	0.14	0.13	0.12	0.15	0.12	0.11	0.14
σ_a	0.47	0.42	0.54	0.44	0.39	0.49	0.48	0.43	0.55	0.44	0.40	0.50	0.47	0.42	0.54	0.49	0.44	0.56
<i>Information friction parameters</i>																		
$\bar{k}_c$	0.23	0.17	0.29	0.60	0.56	0.65	0.53	0.49	0.58	0.25	0.19	0.31	0.25	0.18	0.31	0.24	0.19	0.31
$\bar{k}_i$	0.46	0.42	0.51	0.57	0.54	0.59	0.42	0.39	0.46	0.59	0.57	0.62	0.46	0.42	0.51	0.47	0.43	0.51
$\bar{k}_g$	0.52	0.49	0.56	0.66	0.59	0.72	0.55	0.52	0.59	0.57	0.53	0.63	0.53	0.50	0.56	0.54	0.51	0.57
$\bar{k}_p$	0.33	0.25	0.42	0.06	0.04	0.13	0.14	0.07	0.25	0.23	0.16	0.33	0.33	0.26	0.44	0.25	0.17	0.37
$\bar{k}_w$	0.37	0.23	0.53	0.58	0.32	0.76	0.56	0.27	0.80	0.41	0.20	0.56	0.39	0.23	0.53	0.42	0.26	0.71
$\bar{k}_r$	0.72	0.59	0.82	0.50	0.45	0.55	0.29	0.21	0.38	0.64	0.56	0.73	0.73	0.59	0.83	0.68	0.50	0.78
$\bar{k}_a$	0.01	0.00	0.01	0.59	0.47	0.69	0.07	0.06	0.09	0.08	0.07	0.11	0.01	0.00	0.07	0.06	0.05	0.08
<i>Measurement errors standard deviation</i>																		
σ_{ϵ}^{ne}	0.30	0.26	0.34	0.32	0.29	0.37	0.30	0.27	0.34	0.30	0.27	0.34	0.30	0.27	0.34	0.33	0.30	0.38
σ_{ϵ}^{de}	0.20	0.19	0.23	0.30	0.27	0.34	0.32	0.29	0.37	0.20	0.19	0.23	0.20	0.19	0.23	0.21	0.19	0.23
σ_{ϵ}^{ne}	1.24	1.11	1.39	1.39	1.25	1.56	1.19	1.09	1.35	1.34	1.22	1.50	1.24	1.12	1.41	1.22	1.11	1.38
σ_{ϵ}^{ne}	0.51	0.46	0.59	0.46	0.42	0.52	0.47	0.42	0.52	0.45	0.41	0.52	0.52	0.46	0.61	0.45	0.41	0.52
σ_{ϵ}^{ne}	0.29	0.26	0.35	0.37	0.33	0.42	0.42	0.37	0.49	0.27	0.24	0.32	0.29	0.26	0.36	0.29	0.25	0.35
Marginal Likelihood	-1381.55			-1464.28			-1474.80			-1374.97			-1366.29			-1389.40		
Number of MH Draws	800k			400k			400k			400k			400k			400k		
MH Burn-in	50%			50%			50%			50%			50%			50%		

Note: All estimations used the same priors as the baseline. The no-standard-frictions specification shuts down habits and price and wage indexation and replaces investment adjustment costs with capital adjustment costs. For the CAC model (No inv. adj. costs), the prior for s_k'' is a Gamma distribution with mean 4 and deviation of 2.5 (we use 1.5 s'' , see text)

Table E2: Posterior distributions when shutting down each information friction: Complete dataset

Full information for:			Preference Shock			Investment Shock			Gov. expenditure shock			Price markup shock			Wage markup shock			Monetary shock			TFP shock		
Parameters			Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%	Mode	5%	95%
Endogenous propagation parameters																							
χ	3.79	2.62	5.88	1.50	0.83	1.56	0.82	0.42	1.46	2.49	0.55	4.06	0.82	0.45	1.64	4.88	3.39	6.84	0.46	0.25	0.98		
φ	0.98	0.97	0.99	1.00	0.99	1.00	0.99	0.98	1.00	1.00	0.98	1.00	0.98	0.97	0.99	1.00	0.99	1.00	1.00	0.98	1.00		
a''	0.07	0.07	0.36	0.59	0.40	0.81	0.39	0.26	0.66	0.09	0.08	0.41	0.26	0.16	0.49	0.07	0.06	0.12	0.32	0.18	0.58		
s''	4.44	3.31	6.08	8.15	7.75	9.69	3.12	2.07	4.85	3.65	2.63	5.17	3.88	2.55	5.20	5.22	4.02	6.85	4.27	3.71	5.53		
ϕ_y	0.04	0.02	0.09	0.02	0.01	0.03	0.39	0.28	0.63	0.29	0.20	0.42	0.28	0.21	0.42	0.08	0.04	0.17	0.28	0.19	0.46		
ϕ_π	3.29	2.72	4.19	0.40	0.30	0.56	1.16	0.61	1.71	2.61	1.12	3.40	1.59	1.22	2.07	3.33	2.76	4.15	1.09	0.67	2.13		
ϕ_r	0.89	0.85	0.91	0.96	0.95	0.97	0.91	0.80	0.94	0.90	0.86	0.93	0.90	0.86	0.93	0.89	0.85	0.91	0.94	0.91	0.96		
t_p	0.02	0.01	0.09	0.99	0.99	1.00	0.11	0.04	0.60	0.03	0.01	0.10	0.70	0.23	0.79	0.03	0.01	0.10	0.05	0.02	0.17		
t_w	0.78	0.40	0.91	0.93	0.82	0.97	0.28	0.09	0.66	0.48	0.14	0.82	0.32	0.12	0.73	0.68	0.25	0.86	0.24	0.09	0.64		
ξ_p	0.83	0.79	0.86	0.96	0.95	0.96	0.94	0.92	0.96	0.80	0.77	0.94	0.96	0.95	0.97	0.81	0.77	0.84	0.96	0.94	0.97		
ξ_w	0.82	0.78	0.85	0.83	0.81	0.85	0.87	0.82	0.90	0.77	0.72	0.89	0.84	0.77	0.88	0.76	0.71	0.80	0.87	0.82	0.90		
Steady-state parameters																							
π_{ss}	2.91	2.39	3.31	1.35	1.02	1.92	1.82	1.34	2.42	1.95	1.45	2.64	2.03	1.57	2.76	2.60	1.85	3.06	1.94	1.42	2.54		
r_{ss}	5.83	5.14	6.55	8.00	7.34	9.29	5.78	5.10	6.41	5.83	5.07	6.65	5.36	4.88	6.02	5.96	5.25	6.82	6.08	5.49	6.72		
γ_{ss}	0.30	0.24	0.35	0.45	0.43	0.48	0.47	0.44	0.49	0.47	0.44	0.50	0.48	0.46	0.50	0.37	0.31	0.43	0.45	0.43	0.47		
Structural shocks parameters																							
ρ_c	0.21	0.12	0.31	0.14	0.08	0.22	0.16	0.09	0.24	0.15	0.08	0.23	0.17	0.09	0.25	0.14	0.08	0.23	0.16	0.09	0.24		
ρ_i	0.51	0.43	0.60	0.30	0.19	0.39	0.40	0.31	0.48	0.42	0.35	0.50	0.40	0.34	0.48	0.40	0.34	0.48	0.49	0.41	0.55		
ρ_g	0.98	0.96	0.98	0.99	0.99	0.99	0.98	0.97	0.99	0.97	0.96	0.98	0.98	0.96	0.98	0.98	0.96	0.99	0.98	0.97	0.99		
ρ_p	0.86	0.80	0.90	0.04	0.02	0.12	0.46	0.18	0.56	0.97	0.64	0.99	0.10	0.04	0.36	0.82	0.76	0.88	0.47	0.38	0.60		
ρ_w	0.17	0.08	0.25	0.42	0.32	0.58	0.32	0.22	0.46	0.25	0.15	0.44	0.53	0.36	0.70	0.20	0.10	0.29	0.30	0.20	0.45		
ρ_r	0.40	0.28	0.52	0.51	0.39	0.60	0.86	0.42	0.90	0.35	0.23	0.50	0.41	0.28	0.56	0.22	0.14	0.31	0.46	0.33	0.58		
ρ_a	0.92	0.86	0.97	0.98	0.98	0.99	0.99	0.99	1.00	1.00	0.96	1.00	1.00	0.99	1.00	0.99	0.94	1.00	0.99	0.93	0.99		
σ_c	0.26	0.24	0.30	1.08	0.91	1.44	0.98	0.80	1.38	1.04	0.84	1.45	1.05	0.86	1.52	0.99	0.81	1.40	1.02	0.81	1.40		
σ_i	2.35	2.00	2.83	1.52	1.35	1.72	2.34	2.01	2.88	2.85	2.41	3.46	3.02	2.61	3.64	2.60	2.25	3.09	3.06	2.56	3.62		
σ_g	0.61	0.54	0.71	0.45	0.39	0.52	0.59	0.52	0.67	0.63	0.56	0.73	0.63	0.57	0.73	0.63	0.56	0.73	0.58	0.51	0.67		
σ_p	0.24	0.17	0.36	0.37	0.30	0.44	0.44	0.32	0.85	0.05	0.04	0.07	0.40	0.32	0.55	0.26	0.18	0.38	0.44	0.30	0.92		
σ_w	0.77	0.59	1.30	0.67	0.46	1.12	0.68	0.44	1.56	0.72	0.44	1.22	0.26	0.21	0.33	0.90	0.70	1.71	0.47	0.38	1.29		
σ_r	0.13	0.12	0.15	0.13	0.12	0.15	0.12	0.11	0.14	0.13	0.11	0.14	0.12	0.11	0.14	0.14	0.12	0.15	0.12	0.11	0.14		
σ_a	0.46	0.41	0.53	0.50	0.45	0.58	0.46	0.42	0.54	0.47	0.42	0.54	0.49	0.44	0.58	0.47	0.42	0.54	0.54	0.47	0.62		
Information friction parameters																							
$\bar{k}_c$	-	-	-	0.24	0.18	0.28	0.25	0.20	0.32	0.24	0.19	0.30	0.24	0.18	0.30	0.25	0.19	0.31	0.25	0.19	0.31		
$\bar{k}_i$	0.49	0.44	0.55	-	-	-	0.59	0.53	0.66	0.48	0.44	0.53	0.47	0.43	0.51	0.48	0.43	0.52	0.47	0.43	0.51		
$\bar{k}_g$	0.55	0.52	0.60	0.78	0.69	0.89	-	-	-	0.53	0.50	0.57	0.54	0.51	0.57	0.53	0.50	0.56	0.53	0.50	0.57		
$\bar{k}_p$	0.34	0.27	0.45	0.47	0.40	0.55	0.21	0.14	0.36	-	-	-	0.38	0.26	0.46	0.35	0.28	0.48	0.21	0.12	0.30		
$\bar{k}_w$	0.44	0.30	0.60	0.41	0.29	0.57	0.36	0.23	0.69	0.50	0.34	0.72	-	-	-	0.38	0.24	0.53	0.53	0.28	0.82		
$\bar{k}_r$	0.77	0.65	0.85	0.71	0.64	0.78	0.59	0.48	0.74	0.81	0.62	0.89	0.68	0.52	0.78	-	-	-	0.62	0.49	0.73		
$\bar{k}_a$	0.51	0.06	0.68	0.03	0.02	0.04	0.06	0.04	0.08	0.05	0.02	0.08	0.06	0.05	0.08	0.01	0.00	0.05	-	-	-		
Measurement errors standard deviation																							
$\sigma_{\bar{w}}^{me}$	0.36	0.33	0.41	0.33	0.29	0.38	0.51	0.42	0.59	0.34	0.31	0.39	0.33	0.30	0.38	0.31	0.27	0.35	0.32	0.29	0.37		
$\sigma_{\bar{d}}^{me}$	0.43	0.39	0.48	0.20	0.19	0.23	0.20	0.18	0.23	0.21	0.18	0.23	0.20	0.19	0.23	0.21	0.19	0.23	0.21	0.19	0.23		
$\sigma_{\bar{a}}^{me}$	1.33	1.20	1.51	1.78	1.59	2.01	1.41	1.28	1.64	1.27	1.16	1.45	1.22	1.11	1.39	1.28	1.15	1.44	1.23	1.11	1.39		
$\sigma_{\bar{r}}^{me}$	0.56	0.50	0.64	0.54	0.48	0.61	0.45	0.40	0.51	0.74	0.66	0.84	0.45	0.41	0.52	0.56	0.50	0.64	0.47	0.42	0.54		
σ_r^{me}	0.28	0.25	0.33	0.22	0.20	0.25	0.29	0.25	0.38	0.27	0.24	0.32	0.28	0.24	0.34	0.29	0.25	0.34	0.29	0.25	0.34		
Marginal Likelihood																							
-1470.95 -1456.35 -1430.97 -1430.03 -1399.56																							

Note: All estimations used the same priors as the baseline and used 400k draws and 50% burn-in.

F Higher-order truncation

In the baseline estimation reported in Section 4, we truncate the hierarchy of average higher-order expectations at order $\bar{k} = 6$. To assess the sensitivity of the results to this choice, we re-estimate the DI model on the complete dataset using $\bar{k} = 12$.

The marginal likelihood under $\bar{k} = 12$ is essentially unchanged relative to the baseline, indicating that the lower-order truncation already provides an accurate approximation to the equilibrium policy functions. Most posterior estimates are also similar across the two truncation orders, as reported in Table F1.

The main exceptions are the following. Two standard frictions become weaker: the investment adjustment cost parameter s'' falls from 4.83 to 3.35, and wage indexation ι_w falls from 0.76 to 0.28, both consistent with the view that information frictions can substitute for standard inertial mechanisms. A few other parameters also move: the Taylor-rule response to inflation declines from 3.02 to 1.48 (while the response to output rises from 0.06 to 0.31), the Calvo price parameter ξ_p increases from 0.84 to 0.95, and the persistence of the price-markup shock ρ_p falls from 0.74 to 0.43. These changes leave the marginal likelihood, and the model's fit, essentially unchanged.

G Log-linearized households' problem

G.1 Detrending procedure

Before deriving the log-linearized equations, we discuss the detrending procedure.

All real variables grow along with the productivity trend, so they are detrended as follows: $\hat{Z}_t = \frac{Z_t}{\gamma^t}$, for any real variable Z_t . Nominal variables grow along with the price level P_t , hence they are stationarized using the following procedure: $\hat{Z}_t = \frac{Z_t}{P_t}$.³⁵

One exception is that optimal prices and wages are detrended by dividing them by the price level of the last period: $\hat{P}_{i,t}^* = P_{i,t}^*/P_{t-1}$, while $\hat{W}_{h,t}^* = W_{h,t}^*/\gamma^t P_{t-1}$.³⁶

Stationary variables are transformed by taking their log-deviation to steady-state value as follows: $z_t = \log(Z_t/\bar{Z})$, for any stationary variable Z_t . Thus, lower case variables denote log-deviation from steady-state of the upper case variables.

³⁵One exception is the Lagrange multiplier of the budget constraint, Λ_t , that must be normalized to $\hat{\Lambda}_t = \Lambda_t \gamma^t P_t$, since it reflects the marginal utility of an additional unit of money, which declines as consumption grows at rate γ and the price level rises.

³⁶This simplifies the derivation as $E_{h,t}[\hat{W}_{h,t}^*] = \hat{W}_{h,t}^*$ and $E_{i,t}[\hat{P}_{i,t}^*] = \hat{P}_{i,t}^*$ since agents observe P_{t-1} . This would not be true if we detrended them using P_t .

Table F1: Posterior distributions: $\bar{k} = 6$ vs. $\bar{k} = 12$ truncation (complete dataset)

Parameters	Prior			Baseline ($\bar{k}=6$)			$\bar{k}=12$		
	Dist.	Mean	Std. Dev.	Mode	5%	95%	Mode	5%	95%
<i>Endogenous propagation parameters</i>									
χ	$\mathcal{G}$	2.00	0.75	1.51	0.92	2.14	1.11	0.63	1.40
φ	$\mathcal{B}$	0.50	0.20	1.00	0.99	1.00	0.99	0.97	0.99
a''	$\mathcal{G}$	0.20	0.10	0.06	0.06	0.22	0.22	0.14	0.44
s''	$\mathcal{G}$	4.00	1.50	4.83	3.62	6.80	3.35	2.65	5.00
ϕ_y	$\mathcal{G}$	0.20	0.10	0.06	0.04	0.11	0.31	0.22	0.44
ϕ_π	$\mathcal{G}$	1.50	0.50	3.02	2.43	3.88	1.48	1.09	1.97
ϕ_r	$\mathcal{B}$	0.50	0.20	0.87	0.84	0.90	0.91	0.87	0.93
ι_p	$\mathcal{B}$	0.50	0.20	0.03	0.01	0.13	0.17	0.07	0.70
ι_w	$\mathcal{B}$	0.50	0.20	0.76	0.37	0.91	0.28	0.10	0.67
ξ_p	$\mathcal{B}$	0.67	0.05	0.84	0.80	0.87	0.95	0.92	0.96
ξ_w	$\mathcal{B}$	0.75	0.05	0.81	0.78	0.85	0.87	0.81	0.90
<i>Steady-state parameters</i>									
π_{ss}	$\mathcal{G}$	2.62	0.50	2.07	1.56	2.72	1.93	1.48	2.58
r_{ss}	$\mathcal{G}$	5.15	0.50	6.58	5.83	7.62	5.35	4.80	6.02
γ_{ss}	$\mathcal{G}$	0.50	0.10	0.36	0.31	0.41	0.48	0.46	0.50
<i>Structural shocks parameters</i>									
ρ_c	$\mathcal{B}$	0.50	0.20	0.14	0.07	0.21	0.17	0.09	0.24
ρ_i	$\mathcal{B}$	0.50	0.20	0.42	0.33	0.50	0.41	0.34	0.48
ρ_g	$\mathcal{B}$	0.50	0.20	0.97	0.95	0.99	0.98	0.96	0.99
ρ_p	$\mathcal{B}$	0.50	0.20	0.74	0.65	0.81	0.43	0.12	0.52
ρ_w	$\mathcal{B}$	0.50	0.20	0.17	0.09	0.25	0.34	0.25	0.49
ρ_r	$\mathcal{B}$	0.50	0.20	0.44	0.31	0.57	0.41	0.28	0.56
ρ_a	$\mathcal{B}$	0.50	0.20	1.00	1.00	1.00	1.00	0.99	1.00
σ_c	$\mathcal{IG}$	0.50	1.00	1.06	0.87	1.52	1.02	0.84	1.44
σ_i	$\mathcal{IG}$	2.00	1.00	2.73	2.37	3.26	3.00	2.61	3.62
σ_g	$\mathcal{IG}$	0.20	1.00	0.55	0.49	0.65	0.63	0.56	0.73
σ_p	$\mathcal{IG}$	0.20	1.00	0.26	0.20	0.41	0.42	0.29	0.66
σ_w	$\mathcal{IG}$	0.50	1.00	0.94	0.72	1.76	0.61	0.40	1.30
σ_r	$\mathcal{IG}$	0.20	1.00	0.13	0.12	0.15	0.12	0.11	0.14
σ_a	$\mathcal{IG}$	0.20	1.00	0.47	0.42	0.54	0.48	0.44	0.56
<i>Information friction parameters</i>									
$\bar{k}_c$	$\mathcal{B}$	0.50	0.20	0.23	0.17	0.29	0.25	0.19	0.31
$\bar{k}_i$	$\mathcal{B}$	0.50	0.20	0.46	0.42	0.51	0.47	0.44	0.52
$\bar{k}_g$	$\mathcal{B}$	0.50	0.20	0.52	0.49	0.56	0.53	0.51	0.57
$\bar{k}_p$	$\mathcal{B}$	0.50	0.20	0.33	0.25	0.42	0.26	0.19	0.40
$\bar{k}_w$	$\mathcal{B}$	0.50	0.20	0.37	0.23	0.53	0.50	0.27	0.73
$\bar{k}_r$	$\mathcal{B}$	0.50	0.20	0.72	0.59	0.82	0.68	0.52	0.79
$\bar{k}_a$	$\mathcal{B}$	0.50	0.20	0.01	0.00	0.01	0.06	0.04	0.08
<i>Measurement errors standard deviation</i>									
σ_{dy}^{me}	$\mathcal{IG}$	0.10	0.02	0.30	0.26	0.34	0.33	0.30	0.38
σ_{dc}^{me}	$\mathcal{IG}$	0.07	0.02	0.20	0.19	0.23	0.21	0.19	0.23
σ_{di}^{me}	$\mathcal{IG}$	0.50	0.10	1.24	1.11	1.39	1.24	1.12	1.39
σ_π^{me}	$\mathcal{IG}$	0.15	0.04	0.51	0.46	0.59	0.45	0.41	0.52
σ_r^{me}	$\mathcal{IG}$	0.15	0.04	0.29	0.26	0.35	0.28	0.24	0.34
Marginal Likelihood				-1381.55			-1384.59		
Number of MH Draws				800k			400k		
MH Burn-in				50%			50%		

Note: Both truncations are estimated on the complete dataset (macroeconomic and expectations data) and same priors.

G.2 Euler Equation

We first detrend and log-linearize equations (G.1) and (G.2) to get the usual Euler equation for consumption:

$$E_{h,t}[\Lambda_{h,t}P_t] = \frac{E_{h,t}[e^{x_t^c}]}{C_{h,t} - \varphi C_{h,t-1}} \quad (\text{G.1})$$

$$E_{h,t}[\Lambda_{h,t}] = \beta E_{h,t}[\Lambda_{h,t+1}R_t]. \quad (\text{G.2})$$

Detrending equation (G.1), we get

$$\begin{aligned} E_{h,t}[\gamma^t \Lambda_{h,t} P_t] &= \frac{E_{h,t}[e^{x_t^c}]}{C_{h,t}/\gamma^t - \varphi C_{h,t-1}/(\gamma \cdot \gamma^{t-1})} \\ E_{h,t}[\hat{\Lambda}_{h,t}] &= \frac{E_{h,t}[e^{x_t^c}]}{\hat{C}_{h,t} - \varphi/\gamma \hat{C}_{h,t-1}} \end{aligned}$$

Log-linearizing the equation above, it follows that

$$\bar{\Lambda}_h(1 + E_{h,t}[\lambda_{h,t}]) = \frac{1 + E_{h,t}[x_t^c]}{\bar{C}_h[1 - \varphi/\gamma + c_{h,t} - \varphi/\gamma c_{h,t-1}]}$$

Using the steady-state result $\bar{\Lambda}_h = [\bar{C}_h(1 - \varphi/\gamma)]^{-1}$ we find

$$(1 + E_{h,t}[\lambda_{h,t}]) \left(1 - \frac{\varphi}{\gamma} + c_{h,t} - \frac{\varphi}{\gamma} c_{h,t-1}\right) = \left(1 - \frac{\varphi}{\gamma}\right) (1 + E_{h,t}[x_t^c])$$

Rearranging terms

$$E_{h,t}[\lambda_{h,t}] = \frac{\varphi/\gamma}{1 - \varphi/\gamma} c_{h,t-1} - \frac{1}{1 - \varphi/\gamma} c_{h,t} + E_{h,t}[x_t^c] \quad (\text{G.3})$$

The detrended version of (G.2) is

$$\begin{aligned} E_{h,t}[\gamma^t \Lambda_{h,t} P_t] &= \frac{\beta}{\gamma} E_{h,t} \left[\gamma^{t+1} \Lambda_{h,t+1} P_{t+1} \frac{P_t}{P_{t+1}} R_t \right] \\ E_{h,t}[\hat{\Lambda}_{h,t}] &= \frac{\beta}{\gamma} E_{h,t} \left[\hat{\Lambda}_{h,t+1} \frac{R_t}{\Pi_{t+1}} \right] \end{aligned}$$

We now log-linearize the equation above

$$\bar{\Lambda}_h(1 + E_{h,t}[\lambda_{h,t}]) = \frac{\beta \bar{R}}{\gamma \bar{\Pi}} \bar{\Lambda}_h E_{h,t} [1 + \lambda_{h,t+1} + r_t - \pi_{t+1}]$$

Using the steady-state result that $\bar{\Pi}/\bar{R} = \beta/\gamma$ and rearranging terms

$$E_{h,t}[\lambda_{h,t}] = E_{h,t}[\lambda_{h,t+1} + r_t - \pi_{t+1}] \quad (\text{G.4})$$

Substituting (G.3) into (G.4) implies

$$\begin{aligned} & \frac{\varphi/\gamma}{1 - \varphi/\gamma} c_{h,t-1} - \frac{1}{1 - \varphi/\gamma} c_{h,t} + E_{h,t}[x_t^c] = \\ & E_{h,t} \left[\frac{\varphi/\gamma}{1 - \varphi/\gamma} c_{h,t} - \frac{1}{1 - \varphi/\gamma} c_{h,t+1} + x_{t+1}^c + r_t - \pi_{t+1} \right] \end{aligned}$$

Rearranging terms

$$c_{h,t} = \frac{\varphi/\gamma}{1 + \varphi/\gamma} c_{h,t-1} + \frac{1}{1 + \varphi/\gamma} E_{h,t}[c_{h,t+1}] - \frac{1 - \varphi/\gamma}{1 + \varphi/\gamma} E_{h,t}[r_t - \pi_{t+1} + (x_{t+1}^c - x_t^c)]$$

Integrating over h , we get the log-linearized Euler equation

$$c_t = \frac{\varphi/\gamma}{1 + \varphi/\gamma} c_{t-1} + \frac{1}{1 + \varphi/\gamma} \int_0^1 E_{h,t}[c_{h,t+1}] dh - \frac{1 - \varphi/\gamma}{1 + \varphi/\gamma} \bar{E}_t[r_t - \pi_{t+1} + (x_{t+1}^c - x_t^c)] \quad (\text{G.5})$$

where $c_t \equiv \int_0^1 c_{h,t} dh$ denotes aggregate consumption. The habit reference is the household's own past consumption $c_{h,t-1}$ (internal habits), and the household does not internalize the effect of current consumption on next period's habit stock, a behavioral-friction assumption that preserves the first-order Euler equation.

G.3 Equilibrium condition for investment

In the baseline model, we assume that investment decisions are subject to an investment adjustment cost. However, in a later exercise, we consider adjustment costs in capital instead. This section derives the investment equilibrium condition for both cases.

Investment adjustment costs. The first-order condition for investment (G.6) with adjustment costs in investment is the following:

$$E_{h,t}[\Lambda_{h,t}P_t] = E_{h,t}[\Phi_{h,t}e^{x_t^i}] \left(1 - S\left(\frac{I_{h,t}}{I_{h,t-1}}\right) - S'\left(\frac{I_{h,t}}{I_{h,t-1}}\right) \frac{I_{h,t}}{I_{h,t-1}} \right) + \beta E_{h,t} \left[\Phi_{h,t+1}e^{x_{t+1}^i} S'\left(\frac{I_{h,t+1}}{I_{h,t}}\right) \left(\frac{I_{h,t+1}}{I_{h,t}}\right)^2 \right] \quad (\text{G.6})$$

We start by detrending the equation above

$$E_{h,t}[\hat{\Lambda}_{h,t}] = E_{h,t}[\hat{\Phi}_{h,t}e^{x_t^i}] \left(1 - S\left(\gamma \frac{\hat{I}_{h,t}}{\hat{I}_{h,t-1}}\right) - S'\left(\gamma \frac{\hat{I}_{h,t}}{\hat{I}_{h,t-1}}\right) \gamma \frac{\hat{I}_{h,t}}{\hat{I}_{h,t-1}} \right) + \frac{\beta}{\gamma} E_{h,t} \left[\hat{\Phi}_{h,t+1}e^{x_{t+1}^i} S'\left(\gamma \frac{\hat{I}_{h,t+1}}{\hat{I}_{h,t}}\right) \left(\gamma \frac{\hat{I}_{h,t+1}}{\hat{I}_{h,t}}\right)^2 \right]$$

Using the definition $Q_{h,t} \equiv E_{h,t}[\hat{\Phi}_{h,t}/\hat{\Lambda}_{h,t}]$, it follows that

$$E_{h,t}[\hat{\Lambda}_{h,t}] = Q_{h,t} E_{h,t}[\hat{\Lambda}_{h,t}e^{x_t^i}] \left(1 - S\left(\gamma \frac{\hat{I}_{h,t}}{\hat{I}_{h,t-1}}\right) - S'\left(\gamma \frac{\hat{I}_{h,t}}{\hat{I}_{h,t-1}}\right) \gamma \frac{\hat{I}_{h,t}}{\hat{I}_{h,t-1}} \right) + \frac{\beta}{\gamma} E_{h,t} \left[Q_{h,t+1} \hat{\Lambda}_{h,t+1} e^{x_{t+1}^i} S'\left(\gamma \frac{\hat{I}_{h,t+1}}{\hat{I}_{h,t}}\right) \left(\gamma \frac{\hat{I}_{h,t+1}}{\hat{I}_{h,t}}\right)^2 \right]$$

We now log-linearize the equation above using the following approximations

$$\begin{aligned} \frac{\hat{I}_{h,t}}{\hat{I}_{h,t-1}} &\approx 1 + i_{h,t} - i_{h,t-1} \\ S(\gamma(1 + i_{h,t} - i_{h,t-1})) &\approx S(\gamma) + \gamma S'(\gamma)(i_{h,t} - i_{h,t-1}) = 0 \\ S'(\gamma(1 + i_{h,t+1} - i_{h,t})) &\approx S'(\gamma) + \gamma S''(\gamma)(i_{h,t+1} - i_{h,t}) = \gamma S''(i_{h,t+1} - i_{h,t}) \end{aligned}$$

The log-linearized version of the equilibrium condition for investment is

$$1 + E_{h,t}[\lambda_{h,t}] = \bar{Q}_h \left(1 + q_{h,t} + E_{h,t}[\lambda_{h,t}] + E_{h,t}[x_t^i] \right) \left(1 - \gamma^2 s''(i_{h,t} - i_{h,t-1})(1 + i_{h,t} - i_{h,t-1}) \right) + \bar{Q}_h \beta \gamma^2 E_{h,t} \left[(1 + \lambda_{h,t+1})(1 + x_{t+1}^i) s''(i_{h,t+1} - i_{h,t})(1 + i_{h,t} - i_{h,t-1})^2 \right]$$

Using the steady-state result $\bar{Q}_h = 1$ and dropping second-order terms off, it follows that

$$E_{h,t}[\lambda_{h,t}] = q_{h,t} + E_{h,t}[\lambda_{h,t}] + E_{h,t}[x_t^i] - \gamma^2 s''(i_{h,t} - i_{h,t-1}) + \beta \gamma^2 s''(E_{h,t}[i_{h,t+1}] - i_{h,t})$$

Rearranging terms, we get

$$i_{h,t} = \left(\frac{1}{1+\beta} \right) i_{h,t-1} + \left(\frac{\beta}{1+\beta} \right) E_{h,t}[i_{h,t+1}] + \left(\frac{1}{s''(1+\beta)\gamma^2} \right) (q_{h,t} + E_{h,t}[x_t^i])$$

Integrating over h yields the aggregate equilibrium condition

$$i_t = \left(\frac{1}{1+\beta} \right) i_{t-1} + \left(\frac{\beta}{1+\beta} \right) \int_0^1 E_{h,t}[i_{h,t+1}] dh + \left(\frac{1}{s''(1+\beta)\gamma^2} \right) (q_t + \bar{E}_t[x_t^i]),$$

where $i_t \equiv \int_0^1 i_{h,t} dh$ and $q_t \equiv \int_0^1 q_{h,t} dh$ denote aggregate investment and the aggregate shadow price of capital, respectively. Finally, we normalize the shock as follows

$$i_t = \left(\frac{1}{1+\beta} \right) i_{t-1} + \left(\frac{\beta}{1+\beta} \right) \int_0^1 E_{h,t}[i_{h,t+1}] dh + \left(\frac{1}{s''(1+\beta)\gamma^2} \right) q_t + \bar{E}_t[\hat{x}_t^i], \quad (\text{G.7})$$

where $\hat{x}_t^i \equiv x_t^i / (s''(1+\beta)\gamma^2)$.

Capital adjustment cost. In the case with capital adjustment costs, the first-order condition for investment is

$$E_{h,t}[\Phi_{h,t} e^{x_t^i}] = E_{h,t}[\Lambda_{h,t} P_t] \left(1 + S' \left(\frac{I_{h,t}}{K_{h,t-1}} \right) \right).$$

Detrending and using the definition $Q_{h,t} \equiv E_{h,t}[\hat{\Phi}_{h,t} / \hat{\Lambda}_{h,t}]$ yields

$$Q_{h,t} E_{h,t}[\hat{\Lambda}_{h,t} e^{x_t^i}] = E_{h,t}[\hat{\Lambda}_{h,t}] \left(1 + S' \left(\gamma \frac{\hat{I}_{h,t}}{\hat{K}_{h,t-1}} \right) \right).$$

We log-linearize the equation above using the following first-order approximation around the steady state

$$S' \left(\gamma \frac{\hat{I}_{h,t}}{\hat{K}_{h,t-1}} \right) \approx S' \left(\gamma \frac{\bar{I}}{\bar{K}} \right) + s'' \left(\gamma \frac{\hat{I}_{h,t}}{\hat{K}_{h,t-1}} - \gamma \frac{\bar{I}}{\bar{K}} \right) \approx s'' \cdot \gamma \frac{\bar{I}}{\bar{K}} (i_{h,t} - k_{h,t-1})$$

Using the steady-state result $\bar{I}_h / \bar{K}_h = 1 - (1 - \delta) / \gamma$, it follows that

$$q_{h,t} + E_{h,t}[x_t^i] = s'' (\gamma - (1 - \delta)) (i_{h,t} - k_{h,t-1})$$

Rewriting in terms of investment yields

$$i_{h,t} = k_{h,t-1} + \left(\frac{1}{s''(\gamma - (1 - \delta))} \right) q_{h,t} + E_{h,t}[\hat{x}_t^i]$$

after we use the normalized shock $\hat{x}_t^i \equiv \frac{x_t^i}{s''(\gamma - (1 - \delta))}$. Finally, we integrate over h to get

$$i_t = k_{t-1} + \left(\frac{1}{s''(\gamma - (1 - \delta))} \right) q_t + \bar{E}_t[\hat{x}_t^i] \quad (\text{G.8})$$

G.4 Asset pricing equation for capital stock

In the baseline model, we assume that investment decisions are subject to an investment adjustment cost. However, in a later exercise, we consider adjustment costs in capital instead. This section derives the investment equilibrium condition for both cases.

Investment adjustment costs. We use the households' first-order conditions for capital (G.9) below to get an asset pricing equation for capital:

$$E_{h,t}[\Phi_{h,t}] = \beta E_{h,t} \left[\Lambda_{h,t+1} \left(R_{t+1}^k U_{h,t+1} - P_{t+1} a(U_{h,t+1}) \right) \right] + (1 - \delta) \beta E_{h,t} [\Phi_{h,t+1}] \quad (\text{G.9})$$

We detrend all variables in (G.9),

$$E_{h,t} [\hat{\Phi}_{h,t}] = \frac{\beta}{\gamma} E_{h,t} \left[\hat{\Lambda}_{h,t+1} \left(\hat{R}_{t+1}^k U_{h,t+1} - a(U_{h,t+1}) \right) \right] + (1 - \delta) \frac{\beta}{\gamma} E_{h,t} [\hat{\Phi}_{h,t+1}]$$

Using the definition $Q_{h,t} \equiv E_{h,t}[\Phi_{h,t}/\Lambda_{h,t}P_t]$, it follows that

$$Q_{h,t} E_{h,t} [\hat{\Lambda}_{h,t}] = \frac{\beta}{\gamma} E_{h,t} \left[\hat{\Lambda}_{h,t+1} \left(\hat{R}_{t+1}^k U_{h,t+1} - a(U_{h,t+1}) \right) \right] + (1 - \delta) \frac{\beta}{\gamma} E_{h,t} [Q_{h,t+1} \hat{\Lambda}_{h,t+1}].$$

Note that $Q_{h,t}$ can be taken outside the expectation operator $E_{h,t}[\cdot]$ by the Law of Iterated Expectations.

We start log-linearizing using the following approximation

$$a(U_{h,t+1}) \approx a(\bar{U}_h) + a'(\bar{U}_h)(U_{h,t+1} - \bar{U}_h) = \bar{R}^k(U_{h,t+1} - 1) \approx \bar{R}^k u_{h,t+1}$$

and use the following steady-state results to simplify the equation: $\bar{Q} = 1$, $\bar{U} = 1$, $\bar{R}^k =$

$\frac{\gamma}{\beta} - (1 - \delta)$. It follows that

$$1 + q_{h,t} + E_{h,t}[\lambda_{h,t}] = \left(1 - \frac{(1 - \delta)\beta}{\gamma}\right) \left(1 + E_{h,t}[\lambda_{h,t+1} + r_{t+1}^k]\right) + \left(\frac{(1 - \delta)\beta}{\gamma}\right) (1 + E_{h,t}[q_{h,t+1} + \lambda_{h,t+1}])$$

Simplifying further yields

$$q_{h,t} = \left(1 - \frac{(1 - \delta)\beta}{\gamma}\right) E_{h,t}[r_{t+1}^k] + \left(\frac{(1 - \delta)\beta}{\gamma}\right) E_{h,t}[q_{h,t+1}] + E_{h,t}[\lambda_{h,t+1}] - \lambda_{h,t}$$

Plugging the log-linearized version of the first-order condition for bonds (G.4) into the expression above, it follows that

$$q_{h,t} = \left(\frac{\beta(1 - \delta)}{\gamma}\right) E_{h,t}[q_{h,t+1}] + \left(1 - \frac{\beta(1 - \delta)}{\gamma}\right) E_{h,t}[r_{t+1}^k] - E_{h,t}[r_t - \pi_{t+1}]$$

Integrating over h , we finally get the aggregate asset pricing equation for capital

$$q_t = \left(\frac{\beta(1 - \delta)}{\gamma}\right) \int_0^1 E_{h,t}[q_{h,t+1}] dh + \left(1 - \frac{\beta(1 - \delta)}{\gamma}\right) \bar{E}_t[r_{t+1}^k] - \bar{E}_t[r_t - \pi_{t+1}] \quad (\text{G.10})$$

Capital adjustment cost. In the case with capital adjustment costs, the first-order condition for capital is

$$E_{h,t}[\Phi_{h,t}] = \beta E_{h,t}[\Lambda_{h,t+1} (R_{k,t+1} U_{h,t+1} - P_{t+1} a(U_{h,t+1}))] + \beta E_{h,t} \left[\Phi_{h,t+1} \left[1 - \delta - S_k \left(\frac{I_{h,t+1}}{K_{h,t}} \right) + S'_k \left(\frac{I_{h,t+1}}{K_{h,t}} \right) \frac{I_{h,t+1}}{K_{h,t}} \right] \right]$$

Detrending the equation above yields

$$E_{h,t}[\hat{\Phi}_{h,t}] = \frac{\beta}{\gamma} E_{h,t} \left[\hat{\Lambda}_{h,t+1} \left(\hat{R}_{k,t+1} U_{h,t+1} - a(U_{h,t+1}) \right) \right] + \frac{\beta}{\gamma} E_{h,t} \left[\hat{\Phi}_{h,t+1} \left[1 - \delta - S_k \left(\gamma \frac{\hat{I}_{h,t+1}}{\hat{K}_{h,t}} \right) + S'_k \left(\gamma \frac{\hat{I}_{h,t+1}}{\hat{K}_{h,t}} \right) \gamma \frac{\hat{I}_{h,t+1}}{\hat{K}_{h,t}} \right] \right]$$

Using the definition $Q_{h,t} \equiv E_{h,t}[\Phi_{h,t}/\Lambda_{h,t}P_t]$, it follows that

$$Q_{h,t}E_{h,t}[\hat{\Lambda}_{h,t}] = \frac{\beta}{\gamma}E_{h,t}[\hat{\Lambda}_{h,t+1}(\hat{R}_{t+1}^k U_{h,t+1} - a(U_{h,t+1}))] \\ + \frac{\beta}{\gamma}E_{h,t}\left[Q_{h,t+1}\hat{\Lambda}_{h,t+1}\left[1 - \delta - S_k\left(\gamma\frac{\hat{I}_{h,t+1}}{\hat{K}_{h,t}}\right) + S'_k\left(\gamma\frac{\hat{I}_{h,t+1}}{\hat{K}_{h,t}}\right)\gamma\frac{\hat{I}_{h,t+1}}{\hat{K}_{h,t}}\right]\right].$$

We start log-linearizing the equation above using the following first-order approximations around the steady state:

$$S_k\left(\gamma\frac{\hat{I}_{h,t+1}}{\hat{K}_{h,t}}\right) \approx 0 \\ S'_k\left(\gamma\frac{\hat{I}_{h,t+1}}{\hat{K}_{h,t}}\right) \approx S'_k\left(\gamma\frac{\bar{I}}{\bar{K}}\right) + s''_k\left(\gamma\frac{\hat{I}_{h,t+1}}{\hat{K}_{h,t}} - \gamma\frac{\bar{I}}{\bar{K}}\right) \approx s''_k \cdot \gamma\frac{\bar{I}}{\bar{K}}(i_{h,t+1} - k_{h,t}) \\ \frac{\hat{I}_{h,t+1}}{\hat{K}_{h,t}} \approx \frac{\bar{I}}{\bar{K}}(1 + i_{h,t+1} - k_{h,t})$$

We also use the same first-order approximation for utilization capacity and the same steady-state results as before to get

$$q_{h,t} + E_{h,t}[\lambda_{h,t}] = \left(1 - \frac{(1-\delta)\beta}{\gamma}\right)(E_{h,t}[\lambda_{h,t+1} + r_{t+1}^k]) + \\ \left(\frac{(1-\delta)\beta}{\gamma}\right)(E_{h,t}[q_{h,t+1} + \lambda_{h,t+1}]) + \beta\gamma s''\left(1 - \frac{1-\delta}{\gamma}\right)^2 E_{h,t}[i_{h,t+1} - k_{h,t}]$$

Using the log-linearized version of the first-order condition for bonds (G.4), it follows that

$$q_{h,t} = \left(1 - \frac{(1-\delta)\beta}{\gamma}\right)E_{h,t}[r_{t+1}^k] + \\ \left(\frac{(1-\delta)\beta}{\gamma}\right)E_{h,t}[q_{h,t+1}] + \beta\gamma s''\left(1 - \frac{1-\delta}{\gamma}\right)^2 E_{h,t}[i_{h,t+1} - k_{h,t}] - E_{h,t}[r_t - \pi_{t+1}]$$

Integrating over h yields

$$q_t = \left(1 - \frac{(1-\delta)\beta}{\gamma}\right)\bar{E}_t[r_{t+1}^k] + \\ \left(\frac{(1-\delta)\beta}{\gamma}\right)\int_0^1 E_{h,t}[q_{h,t+1}]dh + \beta\gamma s''\left(1 - \frac{1-\delta}{\gamma}\right)^2 \left(\int_0^1 E_{h,t}[i_{h,t+1}]dh - k_t\right) - \bar{E}_t[r_t - \pi_{t+1}] \quad (\text{G.11})$$

Note that, as $\gamma \approx 1$, $(\bar{I}/\bar{K})^2 \approx \delta^2 \approx 0$.

G.5 Law of motion of capital

In the baseline model, we assume that investment decisions are subject to an investment adjustment cost. However, in a later exercise, we consider adjustment costs in capital instead. This section derives the law of motion of capital for both cases.

Investment adjustment costs. The law of motion of capital when there are adjustment costs in investment is the following

$$K_{h,t} = (1 - \delta)K_{h,t-1} + e^{x_t^i}(1 - S(I_{h,t}/I_{h,t-1}))I_{h,t} \quad (\text{G.12})$$

Detrending the equation above yields

$$\hat{K}_{h,t} = \left(\frac{1 - \delta}{\gamma}\right) \hat{K}_{h,t-1} + e^{x_t^i} \left(1 - S\left(\gamma \frac{\hat{I}_{h,t}}{\hat{I}_{h,t-1}}\right)\right) \hat{I}_{h,t}$$

We log-linearize the equation

$$\bar{K}_h(1 + k_{h,t}) = \left(\frac{1 - \delta}{\gamma}\right) \bar{K}_h(1 + k_{h,t-1}) + (1 + x_t^i)(1 - S(\gamma(1 + i_{h,t} - i_{h,t-1}))) \bar{I}_h(1 + i_{h,t})$$

Using the steady-state result $\bar{I}_h/\bar{K}_h = 1 - (1 - \delta)/\gamma$

$$\begin{aligned} 1 + k_{h,t} &= \left(\frac{1 - \delta}{\gamma}\right) (1 + k_{h,t-1}) + (1 + x_t^i)(1 - S(\gamma(1 + i_{h,t} - i_{h,t-1}))) \left(1 - \frac{1 - \delta}{\gamma}\right) (1 + i_{h,t}) \\ k_{h,t} &= \left(\frac{1 - \delta}{\gamma}\right) k_{h,t-1} + \left(1 - \frac{1 - \delta}{\gamma}\right) (i_{h,t} + x_t^i - S(\gamma(1 + i_{h,t} - i_{h,t-1}))) (1 + i_{h,t} + x_t^i) \end{aligned}$$

We use the following first-order approximation around the steady state

$$S(\gamma(1 + i_{h,t} - i_{h,t-1})) \approx S(\gamma) + \gamma S'(\gamma)(i_{h,t} - i_{h,t-1}) = 0,$$

which yields

$$k_{h,t} = \left(\frac{1 - \delta}{\gamma}\right) k_{h,t-1} + \left(1 - \frac{1 - \delta}{\gamma}\right) (i_{h,t} + x_t^i).$$

Using the normalized shock $\hat{x}_t^i \equiv x_t^i/(s''(1 + \beta)\gamma^2)$ and integrating over h , it follows that

$$k_t = \left(\frac{1 - \delta}{\gamma}\right) k_{t-1} + \left(1 - \frac{1 - \delta}{\gamma}\right) i_t + \left[\left(1 - \frac{1 - \delta}{\gamma}\right) s''(1 + \beta)\gamma^2\right] \hat{x}_t^i. \quad (\text{G.13})$$

Capital adjustment costs. The law of motion of capital when there are adjustment costs in capital is the following

$$K_{h,t} = \left(1 - \delta - S_k \left(\frac{I_{h,t}}{K_{h,t-1}} \right)\right) K_{h,t-1} + e^{x_t^i} I_{h,t}$$

Detrending the equation above yields

$$\hat{K}_{h,t} = \frac{1}{\gamma} \left(1 - \delta - S_k \left(\gamma \frac{\hat{I}_{h,t}}{\hat{K}_{h,t-1}} \right)\right) \hat{K}_{h,t-1} + e^{x_t^i} \hat{I}_{h,t}$$

We log-linearize the equation above as follows

$$\bar{K}_h(1 + k_{h,t}) = \frac{1}{\gamma} [1 - \delta - S_k(\gamma(1 + i_{h,t} - k_{h,t-1}))] \bar{K}_h(1 + k_{h,t-1}) + \bar{I}_h(1 + i_{h,t} + x_t^i)$$

Using the steady-state result $\bar{I}_h = (1 - (1 - \delta)/\gamma)\bar{K}_h$, we get

$$k_{h,t} = \frac{1}{\gamma} [1 - \delta - S_k(\gamma(1 + i_{h,t} - k_{h,t-1}))] k_{h,t-1} + \left(1 - \frac{1 - \delta}{\gamma}\right) (i_{h,t} + x_t^i)$$

We use the following first-order approximation around the steady state

$$S_k(\gamma(1 + i_{h,t} - k_{h,t-1})) \approx S_k\left(\gamma \frac{\bar{I}}{\bar{K}}\right) + \gamma \frac{\bar{I}}{\bar{K}} S'_k\left(\gamma \frac{\bar{I}}{\bar{K}}\right) (i_{h,t} - k_{h,t-1}) = 0,$$

which yields

$$k_{h,t} = \left(\frac{1 - \delta}{\gamma}\right) k_{h,t-1} + \left(1 - \frac{1 - \delta}{\gamma}\right) (i_{h,t} + x_t^i)$$

Using the normalized shock $\hat{x}_t^i \equiv \frac{x_t^i}{s''(\gamma - (1 - \delta))}$ and integrating over h , it follows that

$$k_t = \left(\frac{1 - \delta}{\gamma}\right) k_{t-1} + \left(1 - \frac{1 - \delta}{\gamma}\right) i_t + \left[\left(1 - \frac{1 - \delta}{\gamma}\right) s''(\gamma - (1 - \delta))\right] \hat{x}_t^i, \quad (\text{G.14})$$

G.6 Capital utilization level

The optimal value of the capital utilization rate solves

$$E_{h,t}[R_t^k] = a'(U_{h,t})E_{h,t}[P_t]. \quad (\text{G.15})$$

Based on equation (G.15), we obtain a log-linearized version of the capital utilization function by first dividing both sides of the equation by P_t

$$a'(U_{h,t})E_{h,t} \left[\frac{P_t}{P_{t-1}} \right] = E_{h,t} \left[\hat{R}_t^k \frac{P_t}{P_{t-1}} \right]$$

We now log-linearize each term and use the following approximation

$$a'(U_{h,t}) \approx a'(\bar{U}_h) + a''(\bar{U}_h)(U_{h,t} - \bar{U}_h) = \bar{R}^k + a''u_{h,t}$$

This leads to

$$\begin{aligned} \bar{\Pi}(\bar{R}^k + a''u_{h,t} + \bar{R}^k E_{h,t}[\pi_t]) &= \bar{R}^k \bar{\Pi}(1 + E_{h,t}[r_t^k + \pi_t]) \\ u_{h,t} &= \left(\bar{R}^k / a'' \right) E_{h,t}[r_t^k] \end{aligned}$$

Integrating over h , we have

$$u_t = (\bar{R}^k / a'') \bar{E}_t[r_t^k] \quad (\text{G.16})$$

G.7 Utilized capital level

We log-linearize the capital utilization function $K_{h,t}^u = U_{h,t}K_{h,t-1}$ as follows

$$\begin{aligned} K_{h,t}^u / \gamma^t &= U_{h,t} K_{h,t-1} / (\gamma \cdot \gamma^t) \\ \hat{K}_{h,t}^u &= U_{h,t} \hat{K}_{h,t-1} / \gamma \\ k_{h,t}^u &= u_{h,t} + k_{h,t-1} \end{aligned}$$

Integrating over $h \in [0, 1]$, we get

$$k_t^u = u_t + k_{t-1} \quad (\text{G.17})$$

G.8 Marginal cost of labor

We detrend the marginal rate of substitution between consumption and leisure

$$MRS_{h,t} = L_{h,t}^\chi (C_{h,t} - \varphi C_{h,t-1})$$

as follows

$$\begin{aligned} MRS_{h,t}/\gamma^t &= L_{h,t}^\chi \left(C_{h,t}/\gamma^t - \varphi C_{h,t-1}/(\gamma \cdot \gamma^{t-1}) \right) \\ \widehat{MRS}_{h,t} &= L_{h,t}^\chi \left(\hat{C}_{h,t} - \varphi/\gamma \hat{C}_{h,t-1} \right) \end{aligned}$$

We now log-linearize the variables and use the approximation $(1 + l_{h,t})^\chi \approx 1 + \chi l_{h,t}$

$$M\bar{R}S(1 + mrs_{h,t}) = \bar{L}^\chi \bar{C}(1 + \chi l_{h,t})(1 + c_{h,t} - \varphi/\gamma - \varphi/\gamma c_{h,t-1})$$

Using the fact that $M\bar{R}S = \bar{C}(1 - \varphi/\gamma)\bar{L}^\chi$, we get

$$(1 + mrs_{h,t}) \left(1 - \frac{\varphi}{\gamma} \right) = 1 - \frac{\varphi}{\gamma} + \left(1 - \frac{\varphi}{\gamma} \right) \chi l_{h,t} + c_{h,t} - \frac{\varphi}{\gamma} c_{h,t-1}$$

We rearrange the terms to get

$$mrs_{h,t} = \left(\frac{1}{1 - \varphi/\gamma} \right) \left(c_{h,t} - \frac{\varphi}{\gamma} c_{h,t-1} \right) + \chi l_{h,t}$$

We finally integrate over h to get

$$mrs_t = \left(\frac{1}{1 - \varphi/\gamma} \right) (c_t - \varphi/\gamma c_{t-1}) + \chi l_t \quad (\text{G.18})$$

G.9 Resource constraint

The resource constraint is defined as follows

$$Y_t = C_t + I_t + a(U_t)K_{t-1} + G_t$$

Log-linearizing each term leads to

$$\bar{Y}(1 + y_t) = \bar{C}(1 + c_t) + \bar{I}(1 + i_t) + \bar{K}\bar{R}^k u_t + \bar{G}(1 + g_t)$$

We use the steady-state result that $\bar{Y} = \bar{C} + \bar{I} + \bar{G}$ and divide each side by $\bar{Y}$ to simplify the expression above to

$$y_t = c_y c_t + i_y i_t + \bar{R}^k k_y u_t + g_y g_t,$$

where $c_y = \bar{C}/\bar{Y}$, $i_y = \bar{I}/\bar{Y}$, $k_y = \bar{K}/\bar{Y}$ and $g_y = \bar{G}/\bar{Y}$. We now derive a formula for g_t by noticing that government expenditures follow the rule below

$$\begin{aligned}\frac{G_t}{Y_t} &= g_y + x_t^g g_y (1 + g_t - y_t) = g_y + x_t^g \\ g_t &= y_t + \frac{x_t^g}{g_y}\end{aligned}$$

Plugging the result above into the log-linearized version of the resource constraint implies

$$y_t = \frac{1}{1 - g_y} \left(\frac{\bar{C}}{\bar{Y}} c_t + \frac{\bar{I}}{\bar{Y}} i_t + \frac{\bar{R}^k \bar{K}}{\bar{Y}} u_t + x_t^g \right) \quad (\text{G.19})$$

H Derivation of price- and wage-setting conditions

H.1 Cost minimization problem

$$\begin{aligned}\min_{\{K_{i,t}, L_{i,t}\}} & E_{i,t} [W_t L_{i,t} + R_t^k K_{i,t}] \\ \text{s.t. } & Y_{i,t} = A_t K_{i,t}^\alpha L_{i,t}^{1-\alpha} - \gamma^t \Phi_p, \text{ }^{37}\end{aligned}$$

where A_t follows

$$\log(A_t) = \rho_a \log(A_{t-1}) + \varepsilon_t^a \quad (\text{H.1})$$

The lagrangean is:

$$\min_{\{K_{i,t}, L_{i,t}\}} \mathcal{L} = E_{i,t} [W_t L_{i,t} + R_t^k K_{i,t} - \gamma^t \Phi_p + \mu_{i,t} [Y_{i,t} - A_t K_{i,t}^\alpha L_{i,t}^{1-\alpha}]]$$

The first-order conditions are:

$$\begin{aligned}E_{i,t} [W_t] &= (1 - \alpha) \mu_{i,t} \frac{E_{i,t} [Y_{i,t}]}{L_{i,t}} \\ E_{i,t} [R_t^k] &= \alpha \mu_{i,t} \frac{E_{i,t} [Y_{i,t}]}{K_{i,t}}\end{aligned}$$

Dividing both equations yields:

$$\frac{E_{i,t} [W_t]}{E_{i,t} [R_t^k]} = \frac{1 - \alpha}{\alpha} \frac{K_{i,t}}{L_{i,t}} \quad (\text{H.2})$$

substituting into the production function and solving for $K_{i,t}$ and $L_{i,t}$ as a function of $Y_{i,t}$, it follows that:

$$\begin{aligned} Y_{i,t} &= A_t \left[\frac{\alpha}{1-\alpha} \frac{E_{i,t}[W_t]}{E_{i,t}[R_t^k]} \right]^\alpha L_{i,t} & L_{i,t} &= \left[\frac{1-\alpha}{\alpha} \frac{E_{i,t}[R_t^k]}{E_{i,t}[W_t]} \right]^\alpha \frac{Y_{i,t}}{A_t} \\ Y_{i,t} &= A_t \left[\frac{1-\alpha}{\alpha} \frac{E_{i,t}[R_t^k]}{E_{i,t}[W_t]} \right]^{1-\alpha} K_{i,t} & K_{i,t} &= \left[\frac{\alpha}{1-\alpha} \frac{E_{i,t}[W_t]}{E_{i,t}[R_t^k]} \right]^{1-\alpha} \frac{Y_{i,t}}{A_t} \end{aligned}$$

Substituting both into the total cost function, we get:

$$\begin{aligned} \mathcal{C}(Y_{i,t}) &= \left[W_t \left[\frac{1-\alpha}{\alpha} \frac{E_{i,t}[R_t^k]}{E_{i,t}[W_t]} \right]^\alpha + R_t^k \left[\frac{\alpha}{1-\alpha} \frac{E_{i,t}[W_t]}{E_{i,t}[R_t^k]} \right]^{1-\alpha} \right] \frac{Y_{i,t}}{A_t} - A_t \Phi_p \\ &= \left[\frac{(1-\alpha)W_t E_{i,t}[R_t^k] + \alpha R_t^k E_{i,t}[W_t]}{\alpha^\alpha (1-\alpha)^{1-\alpha} E_{i,t}[W_t]^\alpha E_{i,t}[R_t^k]^{1-\alpha}} \right] \frac{Y_{i,t}}{A_t} - A_t \Phi_p \end{aligned}$$

Finally, the marginal cost is given by:

$$MC_{i,t} \equiv \mathcal{C}'(Y_{i,t}) = \left[\frac{(1-\alpha)W_t E_{i,t}[R_t^k] + \alpha R_t^k E_{i,t}[W_t]}{\alpha^\alpha (1-\alpha)^{1-\alpha} E_{i,t}[W_t]^\alpha E_{i,t}[R_t^k]^{1-\alpha}} \right] \frac{1}{A_t} \quad (\text{H.3})$$

H.2 Price index

The price index is given by

$$P_t = \left[\int_0^1 P_{i,t}^{-\frac{1}{\mu_t}} di \right]^{-\mu_t^p}$$

The firms that do not optimize follow the indexation rule given by

$$X_t = \Pi_{t-1}^{\iota_p} \bar{\Pi}^{1-\iota_p}$$

where $\iota_p \in [0, 1]$.

Observe that

$$\begin{aligned}
P_t^{-\frac{1}{\mu_t^p}} &= \int_0^1 P_{i,t}^{-\frac{1}{\mu_t^p}} di = \int_0^{\xi_p} (X_t P_{i,t-1})^{-\frac{1}{\mu_t^p}} di + \int_{\xi_p}^1 (P_{i,t}^*)^{-\frac{1}{\mu_t^p}} di \\
&= \xi_p X_t^{-\frac{1}{\mu_t^p}} \int_0^1 P_{i,t-1}^{-\frac{1}{\mu_t^p}} di + (1 - \xi_p) \int_0^1 (P_{i,t}^*)^{-\frac{1}{\mu_t^p}} di \\
&= \xi_p (X_t P_{t-1})^{-\frac{1}{\mu_t^p}} + (1 - \xi_p) (P_t^*)^{-\frac{1}{\mu_t^p}}
\end{aligned}$$

where the last equality uses the definition of P_t in period $t - 1$ and uses the definition of aggregate optimal price given by $P_t^* = \left[\int_0^1 (P_{i,t}^*)^{-\frac{1}{\mu_t^p}} di \right]^{-\mu_t^p}$.

Dividing both sides by $P_{t-1}^{-\frac{1}{\mu_t^p}}$ and using the definition of Π_t and X_t :

$$\Pi_t^{-\frac{1}{\mu_t^p}} = \xi_p \left(\Pi_{t-1}^{\iota_p} \bar{\Pi}^{1-\iota_p} \right)^{-\frac{1}{\mu_t^p}} + (1 - \xi_p) \left(\hat{P}_t^* \right)^{-\frac{1}{\mu_t^p}} \quad (\text{H.4})$$

where $\hat{P}_t^* = \frac{P_t^*}{P_{t-1}}$.

H.3 Optimal price

$$\begin{aligned}
&\max_{\{P_{i,t}^*\}} \sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\Lambda_{t,t+s} \left(P_{i,t}^* X_{t,t+s} Y_{t+s|t} - \mathcal{C}(Y_{t+s|t}) \right) \right] \\
\text{s.t. } &Y_{t+s|t} = \left(\frac{P_{i,t}^* X_{t,t+s}}{P_{t+s}} \right)^{-\frac{1+\lambda_{p,t+s}}{\lambda_{p,t+s}}} Y_{t+s} \\
&X_{t,t+s} = \begin{cases} \Pi_{j=1}^s (\Pi_{t+j-1}^{\iota_p} \bar{\Pi}^{1-\iota_p}), & \text{for } s \geq 1 \\ 1, & \text{for } s = 0 \end{cases} \\
&\Lambda_{t,t+s} = \beta^s \frac{\lambda_{t+s}}{\lambda_t} = \beta^s \frac{u_c(C_{t+s})}{u_c(C_t)} \frac{P_t}{P_{t+s}}
\end{aligned}$$

where $Y_{t+s|t}$ denote the output from period $t + s$ given that the last price update was done in period t .

FOC:

$$\sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\Lambda_{t,t+s} \left(X_{t,t+s} Y_{t+s|t} - \left(\frac{1 + \lambda_{p,t+s}}{\lambda_{p,t+s}} \right) P_{i,t}^* X_{t,t+s} \frac{Y_{t+s|t}}{P_{i,t}^*} + \mathcal{C}'(Y_{t+s|t}) \left(\frac{1 + \lambda_{p,t+s}}{\lambda_{p,t+s}} \right) \frac{Y_{t+s|t}}{P_{i,t}^*} \right) \right] = 0$$

$$\sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\Lambda_{t,t+s} Y_{t+s|t} \left(-\frac{X_{t,t+s}}{\lambda_{p,t+s}} + \left(\frac{1 + \lambda_{p,t+s}}{\lambda_{p,t+s}} \right) \frac{MC_{i,t+s}}{P_{i,t}^*} \right) \right] = 0 \quad (\text{H.5})$$

$$\sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\Lambda_{t,t+s} Y_{t+s|t} \frac{X_{t,t+s}}{\lambda_{p,t+s}} \right] = \sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\Lambda_{t,t+s} Y_{t+s|t} \left(\frac{1 + \lambda_{p,t+s}}{\lambda_{p,t+s}} \right) \frac{MC_{i,t+s}}{P_{i,t}^*} \right] \quad (\text{H.6})$$

H.4 Wage-setting problem

$$\begin{aligned} \min_{\{W_{h,t}^*\}} \quad & -E_{h,t} \left[\sum_{s=0}^{\infty} (\beta \xi_w)^s V(L_{h,t+s|t}) \right] \\ \text{s.t.} \quad & L_{h,t+s|t} = \left(\frac{W_{h,t}^* X_{t,t+s}^w}{W_{t+s}} \right)^{-\frac{1+\mu_{w,t+s}}{\mu_{w,t+s}}} L_{t+s} \\ & P_{t+s} C_{h,t+s} + P_{t+s} I_{h,t+s} + B_{h,t+s} + P_{t+s} a(U_{h,t+s}) K_{h,t+s-1} + Q_{t+s+1,t} A_{h,t+s} \\ & = R_{t+s-1} B_{h,t+s-1} + W_{h,t+s} L_{h,t+s|t} + R_{t+s}^k U_{h,t+s} K_{h,t+s-1} + P_{t+s} A_{h,t+s-1} + T_{h,t+s} \\ & W_{h,t+s} = X_{t,t+s}^w W_{h,t}^* \\ & X_{t,t+s}^w = \begin{cases} (\gamma \bar{\Pi}^{1-\iota_w})^s \prod_{j=1}^s (\Pi_{t+j-1}^{\iota_w}) & \text{if } s \geq 1 \\ 1 & \text{if } s = 0. \end{cases} \end{aligned}$$

where $L_{h,t+s|t}$ denotes the labor supply from period $t+s$ given that the last wage update was done in period t .

The Lagrangean of this problem is given by

$$\begin{aligned} \mathcal{L} = \sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[V(L_{h,t+s|t}) + \Lambda_{h,t+s} \left(R_{t+s-1} B_{h,t+s-1} + X_{t,t+s}^w W_{h,t}^* L_{h,t+s|t} + R_{t+s}^k U_{h,t+s} K_{h,t-1} + \right. \right. \\ \left. \left. P_{t+s} A_{h,t+s-1} + T_{h,t+s} - P_{t+s} C_{h,t+s} - P_{t+s} I_{h,t+s} - B_{h,t+s} - P_{t+s} a(U_{h,t+s}) K_{h,t+s-1} - Q_{t+s+1,t} A_{h,t+s} \right) \right] \end{aligned}$$

Note that the Lagrangean multiplier of this problem is equal to the marginal utility of consumption, $\Lambda_{h,t+s}$.

The optimal condition for setting the wage of differentiated labor, $W_{h,t}^*$, is

$$\begin{aligned} 0 &= \sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[\left(\Lambda_{h,t+s} \left(X_{t,t+s}^w L_{h,t+s|t} - \frac{1 + \mu_{t+s}^w}{\mu_{t+s}^w} L_{h,t+s|t} X_{t,t+s}^w \right) + \frac{1 + \mu_{t+s}^w}{\mu_{t+s}^w} V'(L_{h,t+s}) \frac{L_{h,t+s|t}}{W_{h,t}^*} \right) \right] \\ 0 &= \sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[L_{h,t+s|t} \Lambda_{h,t+s} \left(-\frac{X_{t,t+s}^w}{\mu_{t+s}^w} + \frac{1 + \mu_{t+s}^w}{\mu_{t+s}^w} \frac{V'(L_{h,t+s})}{\Lambda_{h,t+s} W_{h,t}^*} \right) \right] \end{aligned}$$

Using the FOC from households' problem and the definition of the rate of marginal substitution between labor and consumption, one can see that

$$\begin{aligned}\frac{V'(L_{h,t+s|t})}{\Lambda_{h,t+s}} &= L_{h,t+s|t}^\chi (C_{h,t+s} - \varphi/\gamma C_{h,t+s-1}) E_{h,t+s}[P_{t+s}] \\ &= MRS_{h,t+s|t} E_{h,t+s}[P_{t+s}].\end{aligned}$$

Thus, the wage-setting equation can be written as

$$\sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[L_{h,t+s|t} \Lambda_{h,t+s} \frac{X_{t,t+s}^w}{\mu_{t+s}^w} \right] = \sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[L_{h,t+s|t} \Lambda_{h,t+s} \frac{1 + \mu_{t+s}^w}{\mu_{t+s}^w} \frac{MRS_{h,t+s|t} E_{h,t+s}[P_{t+s}]}{W_{h,t}^*} \right] \quad (\text{H.7})$$

H.5 Labor packers

There is a continuum of households indexed by $h \in [0, 1]$ who supply differentiated labor services that are imperfect substitutes for other households' labor services. As in [Erceg et al. \(2000\)](#), it is assumed that there are "employment agencies" that combine households' labor supply using the following aggregator

$$L_t = \left(\int_0^1 L_{h,t}^{\frac{1}{1+\mu_t^w}} dh \right)^{1+\mu_t^w}, \quad (\text{H.8})$$

where μ_t^w denote the agency's wage mark-up such that $\log(1 + \mu_t^w) = \log(1 + \bar{\mu}^w) + x_t^w$. In turn, x_t^w follows the process

$$x_t^w = \rho_w x_{t-1}^w + \varepsilon_t^w, \quad \varepsilon_t^w \sim \mathcal{N}(0, \sigma_w^2). \quad (\text{H.9})$$

Agencies pay each household h their wage, $W_{h,t}$, and sell a homogeneous labor service to intermediate firms at a cost of W_t . Therefore, they maximize profits

$$W_t L_t - \int_0^1 W_{h,t} L_{h,t} dh \quad (\text{H.10})$$

subject to the nominal wage index

$$W_t = \left(\int_0^1 (W_{h,t})^{-\frac{1}{\mu_t^w}} dh \right)^{-\mu_t^w}.$$

Thus, the demand for labor for each household h is given by

$$L_{h,t} = \left(\frac{W_{h,t}}{W_t} \right)^{-\frac{1+\mu_t^w}{\mu_t^w}} L_t, \quad (\text{H.11})$$

where $W_t = \left(\int_0^1 (W_{h,t})^{-\frac{1}{\mu_t^w}} dh \right)^{-\mu_t^w}$ is the nominal wage index. Given the optimal prices and the indexation rule, the price level has the following law of motion

$$W_t = \left[\xi_w \left(\gamma \Pi_{t-1}^{\ell_w} \bar{\Pi}^{1-\ell_w} W_{t-1} \right)^{-\frac{1}{\mu_t^w}} + (1 - \xi_w) (W_t^*)^{-\frac{1}{\mu_t^w}} \right]^{-\mu_t^w}. \quad (\text{H.12})$$

where $W_t^* = \int_0^1 W_{h,t}^* dh$

I Log-linearization of price- and wage-setting conditions

Let $z_t \equiv \log(Z_t) - \log(\bar{Z})$ be the log-deviation from the steady-state for any relevant variable Z_t .

I.1 Marginal cost

Equation (H.2)

$$\begin{aligned} \frac{E_{i,t} [W_t]}{E_{i,t} [R_t^k]} &= \frac{1 - \alpha}{\alpha} \frac{K_{i,t}}{L_{i,t}} \\ \frac{E_{i,t} [W_t/P_t]}{E_{i,t} [R_t^k/P_t]} &= \frac{1 - \alpha}{\alpha} \frac{K_{i,t}}{L_{i,t}} \\ \frac{E_{i,t} [\hat{W}_t]}{E_{i,t} [\hat{R}_t^k]} &= \frac{1 - \alpha}{\alpha} \frac{K_{i,t}}{L_{i,t}} \\ \frac{\bar{W}}{\bar{R}^k} \frac{E_{i,t} [e^{\hat{w}_t}]}{E_{i,t} [e^{\hat{r}_t^k}]} &= \frac{1 - \alpha}{\alpha} \frac{\bar{K}}{\bar{L}} e^{k_{i,t} - l_{i,t}} \\ \frac{E_{i,t} [e^{\hat{w}_t}]}{E_{i,t} [e^{\hat{r}_t^k}]} &= e^{k_{i,t} - l_{i,t}} \end{aligned}$$

where the last equality uses the same equation in the steady state.

Log-linearizing each term:

$$\begin{aligned}\frac{E_{i,t} [1 + \hat{w}_t]}{E_{i,t} [1 + \hat{r}_t^k]} &= 1 + k_{i,t} - l_{i,t} \\ 1 + E_{i,t} [\hat{w}_t] &= (1 + k_{i,t} - l_{i,t})(1 + E_{i,t} [\hat{r}_t^k]) \\ E_{i,t} [\hat{w}_t] &= k_{i,t} - l_{i,t} + E_{i,t} [\hat{r}_t^k]\end{aligned}$$

where last equality uses the fact that $x_t y_t \cong 0$ for any pair log-deviation from steady-state, (x_t, y_t) .

Marginal cost (equation H.3)

$$\begin{aligned}MC_{i,t} &= \left[\frac{(1 - \alpha)W_t E_{i,t} [R_t^k] + \alpha R_t^k E_{i,t} [W_t]}{\alpha^\alpha (1 - \alpha)^{1-\alpha} E_{i,t} [W_t]^\alpha E_{i,t} [R_t^k]^{1-\alpha}} \right] \frac{1}{A_t} \\ MC_{i,t}/P_t &= \left[\frac{(1 - \alpha)(W_t/P_t) E_{i,t} [R_t^k/P_t] + \alpha (R_t^k/P_t) E_{i,t} [W_t/P_t]}{\alpha^\alpha (1 - \alpha)^{1-\alpha} E_{i,t} [W_t/P_t]^\alpha E_{i,t} [R_t^k/P_t]^{1-\alpha}} \right] \frac{1}{A_t} \\ \hat{MC}_{i,t} &= \left[\frac{(1 - \alpha)\hat{W}_t E_{i,t} [\hat{R}_t^k] + \alpha \hat{R}_t^k E_{i,t} [\hat{W}_t]}{\alpha^\alpha (1 - \alpha)^{1-\alpha} E_{i,t} [\hat{W}_t]^\alpha E_{i,t} [\hat{R}_t^k]^{1-\alpha}} \right] \frac{1}{A_t} \\ \bar{MC} \exp(mc_{i,t}) &= \left[\frac{(1 - \alpha)\bar{\hat{W}} \bar{\hat{R}}^k \exp(\hat{w}_t + E_{i,t} [\hat{r}_t^k]) + \alpha \bar{\hat{W}} \bar{\hat{R}}^k \exp(\hat{r}_t^k + E_{i,t} [\hat{w}_t])}{\alpha^\alpha (1 - \alpha)^{1-\alpha} \bar{\hat{W}}^\alpha (\bar{\hat{R}}^k)^{1-\alpha} \exp(\alpha E_{i,t} [\hat{w}_t] + (1 - \alpha) E_{i,t} [\hat{r}_t^k])} \right] \frac{1}{\exp(a_t)} \\ \exp(mc_{i,t} + a_t) &= \left[\frac{(1 - \alpha) \exp(\hat{w}_t + E_{i,t} [\hat{r}_t^k]) + \alpha \exp(\hat{r}_t^k + E_{i,t} [\hat{w}_t])}{\exp(\alpha E_{i,t} [\hat{w}_t] + (1 - \alpha) E_{i,t} [\hat{r}_t^k])} \right]\end{aligned}$$

where last equality uses that $\bar{MC} = \frac{\bar{\hat{W}}^{1-\alpha} (\bar{\hat{R}}^k)^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha}}$

Log-linearizing each term:

$$\begin{aligned}1 + mc_{i,t} + a_t &= \frac{(1 - \alpha)(1 + \hat{w}_t + E_{i,t} [\hat{r}_t^k]) + \alpha(1 + \hat{r}_t^k + E_{i,t} [\hat{w}_t])}{1 + \alpha E_{i,t} [\hat{w}_t] + (1 - \alpha) E_{i,t} [\hat{r}_t^k]} \\ (1 + mc_{i,t} + a_t)(1 + \alpha E_{i,t} [\hat{w}_t] + (1 - \alpha) E_{i,t} [\hat{r}_t^k]) &= (1 - \alpha)(1 + \hat{w}_t + E_{i,t} [\hat{r}_t^k]) + \alpha(1 + \hat{r}_t^k + E_{i,t} [\hat{w}_t]) \\ 1 + mc_{i,t} + a_t + \alpha E_{i,t} [\hat{w}_t] + (1 - \alpha) E_{i,t} [\hat{r}_t^k] &= (1 - \alpha)(1 + \hat{w}_t + E_{i,t} [\hat{r}_t^k]) + \alpha(1 + \hat{r}_t^k + E_{i,t} [\hat{w}_t]) \\ mc_{i,t} &= (1 - \alpha)\hat{w}_t + \alpha\hat{r}_t^k - a_t\end{aligned}$$

Note that $mc_{i,t}$ does not depend on i . This is due to the fact that the production function exhibits constant returns to scale, and, as it turns out, the heterogeneity of expectations

does not play a role in the cost minimization problem up to the first-order approximation. Thus, we can simply drop the i such that

$$mc_t = (1 - \alpha)\hat{w}_t + \alpha\hat{r}_t^k - a_t$$

I.2 Production function of intermediate producers

Production function:

$$Y_{i,t} = e^{x_t^a} K_{i,t}^\alpha \left(\gamma^t L_{i,t} \right)^{1-\alpha} - \gamma^t \Phi_p$$

We first detrend the variables to get

$$\hat{Y}_{i,t} = e^{x_t^a} \hat{K}_{i,t}^\alpha L_{i,t}^{1-\alpha} - \Phi_p$$

We move the term Φ_p to the left-hand side of the equation and log-linearize it as follows

$$\log(\hat{Y}_{i,t} + \Phi_p) - \log(\bar{Y}_i + \Phi_p) = x_t^a + \alpha k_{i,t} + (1 - \alpha)l_{i,t}$$

We may rearrange the left-hand side of the expression above to get

$$\log(\hat{Y}_{i,t} + \Phi_p) - \log(\bar{Y}_i + \Phi_p) = \frac{\bar{Y}_i}{\bar{Y}_i + \Phi_p} y_{i,t}$$

We conclude that

$$y_{i,t} = \frac{\bar{Y}_i + \Phi_p}{\bar{Y}_i} (x_t^a + \alpha k_{i,t} + (1 - \alpha)l_{i,t})$$

Integrating over i , we finally get

$$y_t = \frac{\bar{Y} + \Phi_p}{\bar{Y}} (x_t^a + \alpha k_t + (1 - \alpha)l_t) \quad (\text{I.1})$$

I.3 Price index

Price index (equation H.4) requires a different log-linearization rule. Consider $X_t = Y_t^{Z_t}$. Then, $x_t = \bar{Z}(\log(\bar{Y})z_t + y_t)$ Applying this to equation (H.4):

$$\begin{aligned} \Pi_t^{-\frac{1}{\mu_t^p}} &= \xi_p \left(\Pi_{t-1}^{\iota_p} \bar{\Pi}^{1-\iota_p} \right)^{-\frac{1}{\mu_t^p}} + (1 - \xi_p) \left(\hat{P}_t^* \right)^{-\frac{1}{\mu_t^p}} \\ -\frac{1}{\bar{\lambda}_p} (-\log(\bar{\Pi})\mu_t^p + \pi_t) &= -\xi_p \left(\frac{1}{\bar{\lambda}_p} (-\log(\bar{\Pi})\mu_t^p + \iota_p \pi_{t-1}) \right) - (1 - \xi_p) \frac{1}{\bar{\lambda}_p} (-\log(\bar{\Pi})\mu_t^p + \hat{p}_t^*), \end{aligned}$$

which rearranging leads to

$$\pi_t = \xi_p \pi_{t-1} + (1 - \xi_p) \hat{p}_t^* \quad (\text{I.2})$$

I.4 Optimal price

Optimal price (equation H.6):

$$\begin{aligned} \sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\Lambda_{t,t+s} Y_{t+s|t} \frac{X_{t,t+s}}{\lambda_{p,t+s}} \right] &= \sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\Lambda_{t,t+s} Y_{t+s|t} \left(\frac{1 + \lambda_{p,t+s}}{\lambda_{p,t+s}} \right) \frac{MC_{i,t+s}}{P_{i,t}^*} \right] \\ \sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\Lambda_{t,t+s} Y_{t+s|t} \frac{X_{t,t+s}}{\lambda_{p,t+s}} \right] &= \sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\Lambda_{t,t+s} Y_{t+s|t} \left(\frac{1 + \lambda_{p,t+s}}{\lambda_{p,t+s}} \right) \frac{MC_{i,t+s}}{P_{t+s}} \frac{P_{t-1}}{P_{i,t}^*} \frac{P_{t+s}}{P_{t-1}} \right] \\ \sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\Lambda_{t,t+s} Y_{t+s|t} \frac{X_{t,t+s}}{\lambda_{p,t+s}} \right] &= \sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\Lambda_{t,t+s} Y_{t+s|t} \left(\frac{1 + \lambda_{p,t+s}}{\lambda_{p,t+s}} \right) \frac{\hat{M}C_{i,t+s} \Pi_{t-1,t+s}}{\hat{P}_{i,t}^*} \right] \end{aligned}$$

where $\Pi_{t-1,t+s} = \frac{P_{t+s}}{P_{t-1}} = \sum_{j=0}^s \Pi_{t+j}$ and $\hat{P}_{i,t}^* = P_{i,t}^*/P_{t-1}$. If we stationarize $\hat{P}_{i,t}^*$ by P_t , we wouldn't be able to take it as known by firms, since P_t is unknown. Thus, we proceed by dividing by P_{t-1} .

The equation above can be rewritten as

$$\begin{aligned} \sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\Lambda_{t,t+s} Y_{t+s|t} \frac{X_{t,t+s}}{\lambda_{p,t+s}} \right] &= \sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\Lambda_{t,t+s} Y_{t+s|t} \left(\frac{1 + \lambda_{p,t+s}}{\lambda_{p,t+s}} \right) \frac{\hat{M}C_{i,t+s} \Pi_{t-1,t+s}}{\hat{P}_{i,t}^*} \right] \\ \sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\frac{\bar{\Lambda}_s \bar{Y} \bar{X}_s}{\bar{\lambda}_p} \exp \left(\lambda_{t,t+s} + y_{t+s|t} + x_{t,t+s} - \tilde{\lambda}_{p,t+s} \right) \right] &= \\ \sum_{s=0}^{\infty} \xi_p^s E_{i,t} \left[\frac{\bar{\Lambda}_s \bar{Y} (1 + \bar{\lambda}_p) \bar{M}C \bar{\Pi}^{s+1}}{\bar{\lambda}_p \bar{\Pi}} \exp \left(\lambda_{t,t+s} + y_{t+s|t} + x_{t,t+s} + \tilde{\lambda}_{p,t+s} - \tilde{\lambda}_{p,t+s} + \hat{m}c_{i,t+s} + \pi_{t-1,t+s} - \hat{p}_{i,t}^* \right) \right] \end{aligned}$$

where $x_t^p = \log(\mu_t^p) - \log(\bar{\lambda}_p) \approx \log(1 + \mu_t^p) - \log(1 + \bar{\lambda}_p)$.

Note in the steady-state $\bar{\Lambda}_s = \left(\frac{\beta}{\bar{\Pi}}\right)^s$, $\bar{X}_s = \Pi_{j=1}^s(\bar{\Pi}) = \bar{\Pi}^s$ and $\bar{M}C = 1/(1 + \bar{\lambda}_p)$. Thus, the equation above can be written as

$$\begin{aligned} \sum_{s=0}^{\infty} (\beta \xi_p)^s E_{i,t} \left[\exp \left(\lambda_{t,t+s} + y_{t+s|t} + x_{t,t+s} - \tilde{\lambda}_{p,t+s} \right) \right] &= \\ \sum_{s=0}^{\infty} (\beta \xi_p)^s E_{i,t} \left[\exp \left(\lambda_{t,t+s} + y_{t+s|t} + \hat{m}c_{i,t+s} + \pi_{t-1,t+s} - \hat{p}_{i,t}^* \right) \right] \end{aligned}$$

Loglinearizing leads to:

$$\begin{aligned}
\sum_{s=0}^{\infty} (\beta \xi_p)^s E_{i,t} \left[1 + \lambda_{t,t+s} + y_{t+s|t} + x_{t,t+s} - \tilde{\lambda}_{p,t+s} \right] &= \sum_{s=0}^{\infty} (\beta \xi_p)^s E_{i,t} \left[1 + \lambda_{t,t+s} + y_{t+s|t} + \hat{m}c_{i,t+s} \right. \\
&\quad \left. + \pi_{t-1,t+s} - \hat{p}_{i,t}^* \right] \\
\sum_{s=0}^{\infty} (\beta \xi_p)^s E_{i,t} \left[x_{t,t+s} - \tilde{\lambda}_{p,t+s} \right] &= \sum_{s=0}^{\infty} (\beta \xi_p)^s E_{i,t} \left[\hat{m}c_{i,t+s} + \pi_{t-1,t+s} - \hat{p}_{i,t}^* \right] \\
\frac{\hat{p}_{i,t}^*}{1 - \beta \xi_p} &= \sum_{s=0}^{\infty} (\beta \xi_p)^s E_{i,t} \left[\hat{m}c_{i,t+s} + \pi_{t-1,t+s} - x_{t,t+s} + \tilde{\lambda}_{p,t+s} \right] \\
\hat{p}_{i,t}^* &= (1 - \beta \xi_p) \sum_{s=0}^{\infty} (\beta \xi_p)^s E_{i,t} \left[\hat{m}c_{i,t+s} + \pi_{t-1,t+s} - x_{t,t+s} + \tilde{\lambda}_{p,t+s} \right]
\end{aligned}$$

Note that for $s \geq 1$, we have that

$$\begin{aligned}
X_{t,t+s} &= \prod_{j=1}^s (\Pi_{t+j-1}^{\iota_p} \bar{\Pi}^{1-\iota_p}) \\
\bar{\Pi}^s \exp(x_{t,t+s}) &= \bar{\Pi}^s \exp \left(\iota_p \sum_{j=1}^s \pi_{t+j-1} \right)
\end{aligned}$$

Log-linearizing, we find that:

$$x_{t,t+s} = \iota_p \sum_{j=1}^s \pi_{t+j-1} \quad (\text{I.3})$$

and $x_{t,t} = 0$ by the definition of $X_{t,t+s}$. Analogously, we have that

$$\pi_{t-1,t+s} = \sum_{j=0}^s \pi_{t+j} \quad (\text{I.4})$$

Using equations (I.3-I.4) and the fact that the marginal cost does not depend on i , the optimal price is given by

$$\hat{p}_{i,t}^* = (1 - \beta \xi_p) \sum_{s=0}^{\infty} (\beta \xi_p)^s E_{i,t} \left[\hat{m}c_{t+s} + \sum_{j=0}^s \pi_{t+j} - \iota_p \sum_{j=1}^s \pi_{t+j-1} + \tilde{\lambda}_{p,t+s} \right]$$

We now use the usual trick to rewrite $\hat{p}_{i,t}^*$ recursively.

$$\begin{aligned}
\hat{p}_{i,t}^* &= (1 - \beta\xi_p) \sum_{s=0}^{\infty} (\beta\xi_p)^s E_{i,t} \left[\hat{m}c_{t+s} + \sum_{j=0}^s \pi_{t+j} - \iota_p \sum_{j=1}^s \pi_{t+j-1} + \tilde{\lambda}_{p,t+s} \right] \\
&= (1 - \beta\xi_p) E_{i,t} [\hat{m}c_t + \pi_t + x_t^p] \\
&\quad + (1 - \beta\xi_p) \sum_{s=1}^{\infty} (\beta\xi_p)^s E_{i,t} \left[\hat{m}c_{t+s} + \sum_{j=0}^s \pi_{t+j} - \iota_p \sum_{j=1}^s \pi_{t+j-1} + \tilde{\lambda}_{p,t+s} \right] \\
&= (1 - \beta\xi_p) E_{i,t} [\hat{m}c_t + \pi_t + x_t^p] \\
&\quad + (1 - \beta\xi_p) \sum_{s=0}^{\infty} (\beta\xi_p)^{s+1} E_{i,t} \left[\hat{m}c_{i,t+s+1} + \sum_{j=0}^{s+1} \pi_{t+j} - \iota_p \sum_{j=1}^{s+1} \pi_{t+j-1} + \tilde{\lambda}_{p,t+s+1} \right] \\
&= (1 - \beta\xi_p) E_{i,t} [\hat{m}c_t + \pi_t + x_t^p] \\
&\quad + (1 - \beta\xi_p) \beta\xi_p \sum_{s=0}^{\infty} (\beta\xi_p)^s E_{i,t} \left[\hat{m}c_{i,t+s+1} + \pi_t + \sum_{j=1}^{s+1} \pi_{t+j} - \iota_p \left(\pi_t + \sum_{j=2}^{s+1} \pi_{t+j-1} \right) + \tilde{\lambda}_{p,t+s+1} \right] \\
&= (1 - \beta\xi_p) E_{i,t} [\hat{m}c_t + \pi_t + x_t^p] + \beta\xi_p (1 - \iota_p) E_{i,t} [\pi_t] \\
&\quad + (1 - \beta\xi_p) \beta\xi_p \sum_{s=0}^{\infty} (\beta\xi_p)^s E_{i,t} \left[\hat{m}c_{i,t+s+1} + \sum_{j=0}^s \pi_{t+j+1} - \iota_p \sum_{j=1}^s \pi_{t+j} + \tilde{\lambda}_{p,t+s+1} \right]
\end{aligned}$$

where the second equality separates the $s = 0$ term from the summation, and the third redefines the summation counter as $s = s - 1$. The fourth equality separates the first term from the summations inside the brackets, and the fifth redefines the summation counter as $j = j - 1$ and solves the infinite sum for terms that do not depend on s .

Taking the expectation of the optimal price one period ahead leads to

$$\begin{aligned}
E_{i,t}[\hat{p}_{i,t+1}^*] &= (1 - \beta\xi_p) \sum_{s=0}^{\infty} (\beta\xi_p)^s E_{i,t} \left[E_{i,t+1} \left[\hat{m}c_{i,t+s+1} + \sum_{j=0}^s \pi_{t+j+1} - \iota_p \sum_{j=1}^s \pi_{t+j} + \tilde{\lambda}_{p,t+s+1} \right] \right] \\
&= (1 - \beta\xi_p) \sum_{s=0}^{\infty} (\beta\xi_p)^s E_{i,t} \left[\hat{m}c_{i,t+s+1} + \sum_{j=0}^s \pi_{t+j+1} - \iota_p \sum_{j=1}^s \pi_{t+j} + \tilde{\lambda}_{p,t+s+1} \right]
\end{aligned}$$

using the law of iterated expectations (which holds for i -th expectation but not for the average expectation). Thus, the optimal price can be written as

$$\hat{p}_{i,t}^* = (1 - \beta\xi_p) E_{i,t} [\hat{m}c_t + x_t^p] + (1 - \iota_p \beta\xi_p) E_{i,t} [\pi_t] + \beta\xi_p E_{i,t} [\hat{p}_{i,t+1}^*] \quad (\text{I.5})$$

Finally, we need to aggregate the optimal price to find the NKPC. Aggregating $\hat{p}_{i,t}^*$ leads

to

$$\hat{p}_t^* = (1 - \beta\xi_p)\bar{E}_t[\hat{m}c_t + x_t^p] + (1 - \iota_p\beta\xi_p)\bar{E}_t[\pi_t] + \beta\xi_p \int_0^1 E_{i,t}[\hat{p}_{i,t+1}^*] di$$

Note that $\int_0^1 E_{i,t}[\hat{p}_{i,t+1}^*] di \neq \bar{E}_t[\hat{p}_{t+1}^*]$.

Substituting into equation (I.2) leads to the NKPC under heterogeneous information and indexation

$$\pi_t = \xi_p\iota_p\pi_{t-1} + \kappa_p\xi_p\bar{E}_t[\hat{m}c_t + x_t^p] + (1 - \xi_p)(1 - \iota_p\beta\xi_p)\bar{E}_t[\pi_t] + (1 - \xi_p)\beta\xi_p \int_0^1 E_{i,t}[\hat{p}_{i,t+1}^*] di \quad (\text{I.6})$$

where $\kappa_p = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}$. Inflation depends on its own average expectation with coefficient $\psi_p \equiv (1 - \xi_p)(1 - \iota_p\beta\xi_p)$, which governs how fast the dependence on higher-order beliefs decays with the order in equation (28) of the main text.

I.5 Wage index

We can rewrite the wage index $W_t = \left(\int_0^1 (W_{h,t})^{-\frac{1}{\mu_t^w}} dh \right)^{-\mu_t^w}$ as follows

$$\begin{aligned} W_t &= \left[\xi_w \left(\gamma \Pi_{t-1}^{\iota_w} \bar{\Pi}^{1-\iota_w} W_{t-1} \right)^{-\frac{1}{\mu_t^w}} + (1 - \xi_w) (W_t^*)^{-\frac{1}{\mu_t^w}} \right]^{-\mu_t^w} \\ \frac{W_t}{\gamma^t P_t} &= \left[\xi_w \left(\Pi_{t-1}^{\iota_w} \bar{\Pi}^{1-\iota_w} \frac{W_{t-1}}{\gamma^{t-1} P_{t-1}} \frac{P_{t-1}}{P_t} \right)^{-\frac{1}{\mu_t^w}} + (1 - \xi_w) \left(\frac{W_t^*}{\gamma^t P_{t-1}} \frac{P_{t-1}}{P_t} \right)^{-\frac{1}{\mu_t^w}} \right]^{-\mu_t^w} \\ (\hat{W}_t \Pi_t)^{-\frac{1}{\mu_t^w}} &= \xi_w \left(\Pi_{t-1}^{\iota_w} \bar{\Pi}^{1-\iota_w} \hat{W}_{t-1} \right)^{-\frac{1}{\mu_t^w}} + (1 - \xi_w) (\hat{W}_t^*)^{-\frac{1}{\mu_t^w}} \end{aligned}$$

Note that in the steady-state: $(\bar{\hat{W}}\bar{\Pi})^{-\frac{1}{\bar{\mu}^w}} = \xi_w \left(\bar{\Pi}\bar{\hat{W}} \right)^{-\frac{1}{\bar{\mu}^w}} + (1 - \xi_w) \left(\bar{\hat{W}}^* \right)^{-\frac{1}{\bar{\mu}^w}}$, which implies that $\bar{\hat{W}}^* = \bar{\hat{W}}\bar{\Pi}$.

Following the same log-linearization technique from the price index leads to

$$\begin{aligned} -\frac{1}{\bar{\mu}^w}(-\log(\bar{\Pi}\bar{\hat{W}})\mu_t^w + \hat{w}_t + \pi_t) &= -\xi_w \left(\frac{1}{\bar{\mu}^w}(-\log(\bar{\Pi}\bar{\hat{W}})\mu_t^w + \iota_w\pi_{t-1} + \hat{w}_{t-1}) \right) \\ &\quad - (1 - \xi_w) \frac{1}{\bar{\mu}^w}(-\log(\bar{\hat{W}}^*)\mu_t^w + \hat{w}_t^*), \end{aligned}$$

which rearranging leads to

$$\hat{w}_t + \pi_t = \xi_w(\iota_w\pi_{t-1} + \hat{w}_{t-1}) + (1 - \xi_w)\hat{w}_t^* \quad (\text{I.7})$$

where $\hat{w}_t^* = \int_0^1 \hat{w}_{h,t}^* dh$.

I.6 Optimal wage

The optimal condition of the wage setting problem is

$$E_{h,t} \left[\sum_{s=0}^{\infty} (\beta \xi_w)^s \left(\Lambda_{t,t+s} \frac{X_{t,t+s}^w}{\mu_{t+s}^w} - \frac{1 + \mu_{t+s}^w}{\mu_{t+s}^w} \frac{L_{h,t+s}^x}{W_{h,t}^*} \right) L_{h,t+s} \right] = 0 \quad (\text{I.8})$$

It follows from equation (I.8) that

$$\begin{aligned} \sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[L_{h,t+s|t} \Lambda_{h,t+s} \frac{X_{t,t+s}^w}{\mu_{t+s}^w} \right] &= \sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[L_{h,t+s|t} \Lambda_{h,t+s} \frac{1 + \mu_{t+s}^w}{\mu_{t+s}^w} \frac{MRS_{h,t+s|t} E_{h,t+s}[P_{t+s}]}{W_{h,t}^*} \right] \\ \sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[L_{h,t+s|t} \frac{\hat{\Lambda}_{h,t+s}}{\gamma^s} \frac{X_{t,t+s}^w}{\mu_{t+s}^w \Pi_{t-1,t+s}} \right] \\ &= \sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[L_{h,t+s|t} \hat{\Lambda}_{h,t+s} \frac{1 + \mu_{t+s}^w}{\mu_{t+s}^w} \frac{M\hat{R}S_{h,t+s|t} E_{h,t+s}[\Pi_{t-1,t+s}]}{\hat{W}_{h,t}^* \Pi_{t-1,t+s}} \right] \end{aligned}$$

This equation in the steady state implies that

$$\begin{aligned} \sum_{s=0}^{\infty} (\beta \xi_w)^s \bar{L} \frac{\bar{\hat{\Lambda}}}{\gamma^s} \frac{(\gamma \bar{\Pi})^s}{\bar{\mu}^w \bar{\Pi}^s} &= \sum_{s=0}^{\infty} (\beta \xi_w)^s \bar{L} \bar{\hat{\Lambda}} \frac{1 + \bar{\mu}^w}{\bar{\mu}^w} \frac{M\bar{R}S \bar{\Pi}^{s+1}}{\bar{W} \bar{\Pi}^{s+1}} \\ \sum_{s=0}^{\infty} (\beta \xi_w)^s &= \sum_{s=0}^{\infty} (\beta \xi_w)^s \frac{(1 + \bar{\mu}^w) M\bar{R}S}{\bar{W}} \\ \bar{W} &= (1 + \bar{\mu}^w) M\bar{R}S \end{aligned}$$

Now we turn to the log-linearization:

$$\begin{aligned}
& \sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[\exp \left(l_{h,t+s|t} + \hat{\lambda}_{h,t+s} + x_{t,t+s}^w - \pi_{t-1,t+s} - \tilde{\mu}_{t+s}^w \right) \right] \\
&= \sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[\exp \left(l_{h,t+s|t} + \hat{\lambda}_{h,t+s} + m\hat{r}s_{h,t+s|t} + E_{h,t+s}[\pi_{t-1,t+s}] - \hat{w}_{h,t}^* - \pi_{t-1,t+s} \right) \right] \\
&= \sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[1 + l_{h,t+s|t} + \hat{\lambda}_{h,t+s} + x_{t,t+s}^w - \pi_{t-1,t+s} - \tilde{\mu}_{t+s}^w \right] \\
&= \sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[1 + l_{h,t+s|t} + \hat{\lambda}_{h,t+s} + m\hat{r}s_{h,t+s|t} - \hat{w}_{h,t}^* \right] \\
\hat{w}_{h,t}^* &= (1 - \beta \xi_w) \sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[m\hat{r}s_{h,t+s|t} + \pi_{t-1,t+s} - x_{t,t+s}^w + \tilde{\mu}_{t+s}^w \right] \\
\hat{w}_{h,t}^* &= (1 - \beta \xi_w) \sum_{s=0}^{\infty} (\beta \xi_w)^s E_{h,t} \left[m\hat{r}s_{h,t+s|t} + \sum_{j=0}^s \pi_{t+j} - \iota_w \sum_{j=1}^s \pi_{t+j-1} + \tilde{\mu}_{t+s}^w \right]
\end{aligned}$$

where $\tilde{\mu}_t^w = \log(\mu_t^w) - \log(\bar{\mu}^w) \approx \log(1 + \mu_t^w) - \log(1 + \bar{\mu}^w)$.

Using the labor demand (14) at period $t + s$ for a given wage set at t , we have that

$$\begin{aligned}
L_{h,t+s|t} &= \left(\frac{W_{h,t}^* X_{t,t+s}^w}{W_{t+s}} \right)^{-\frac{1+\mu_{t+s}^w}{\mu_{t+s}^w}} L_{t+s} \\
L_{h,t+s|t} &= \left(\frac{\hat{W}_{h,t}^* X_{t,t+s}^w}{\gamma^s \hat{W}_{t+s} \Pi_{t-1,t+s}} \right)^{-\frac{1+\mu_{t+s}^w}{\mu_{t+s}^w}} L_{t+s}
\end{aligned}$$

Since $\bar{X}_s^w = (\gamma \bar{\Pi})^s$, $\bar{\Pi}_{-1,s} = \bar{\Pi}^{s+1}$, and $\bar{W}^* = \bar{W} \bar{\Pi}$, we have that $\bar{L}_h = \bar{L}$. Log-linearizing implies

$$l_{h,t+s|t} = -\frac{1 + \bar{\mu}^w}{\bar{\mu}^w} \left[\hat{w}_{h,t}^* + \iota_w \sum_{j=1}^s \pi_{t+j-1} - \hat{w}_{t+s} - \sum_{j=0}^s \pi_{t+j} \right] + l_{t+s} \quad (\text{I.9})$$

where the first equality uses that $\log \left(\frac{\bar{W}^* \bar{X}_s^w}{\bar{W} \gamma^s \bar{\Pi}^{s+1}} \right) = \log(1) = 0$. and the second uses that, analogously to the indexation scheme for prices, $x_{t,t+s}^w = \iota_w \sum_{j=1}^s \pi_{t+j-1}$.

The MRS at period $t + s$ for a given wage set at t is given by

$$\begin{aligned} m\hat{r}s_{h,t+s|t} &= \left(\frac{1}{1 - \varphi/\gamma} \right) (c_{h,t+s} - \varphi/\gamma c_{h,t+s-1}) + \chi l_{t+s|t} \\ &= \left(\frac{1}{1 - \varphi/\gamma} \right) (c_{h,t+s} - \varphi/\gamma c_{h,t+s-1}) \\ &\quad + \chi \left(-\frac{1 + \bar{\mu}^w}{\bar{\mu}^w} \left[\hat{w}_{h,t}^* + \iota_w \sum_{j=1}^s \pi_{t+j-1} - \hat{w}_{t+s} - \sum_{j=0}^s \pi_{t+j} \right] + l_{t+s} \right) \end{aligned}$$

Substituting the labor-demand expression for $l_{t+s|t}$ into $m\hat{r}s_{h,t+s|t}$ and collecting the wage-choice $\hat{w}_{h,t}^*$ on the left-hand side yields

$$m\hat{r}s_{h,t+s|t} = \widetilde{mrs}_{h,t+s} - \chi\theta_w \left[\hat{w}_{h,t}^* + \iota_w \sum_{j=1}^s \pi_{t+j-1} - \hat{w}_{t+s} - \sum_{j=0}^s \pi_{t+j} \right]$$

where $\theta_w \equiv \frac{1 + \bar{\mu}^w}{\bar{\mu}^w}$ and

$$\widetilde{mrs}_{h,t+s} \equiv \left(\frac{1}{1 - \varphi/\gamma} \right) (c_{h,t+s} - \varphi/\gamma c_{h,t+s-1}) + \chi l_{t+s}.$$

Note that $\widetilde{mrs}_{h,t+s}$ retains individual consumption $c_{h,t+s}$: under incomplete markets, household h 's consumption differs from the aggregate, but its own current and past consumption are in h 's information set at $t + s$. The relative-wage component of labor has been absorbed into the $(1 + \chi\theta_w)$ denominator below; only aggregate labor l_{t+s} remains.

Substituting back into the optimal-wage condition,

$$\hat{w}_{h,t}^* = \frac{(1 - \beta\xi_w)}{1 + \chi\theta_w} \sum_{s=0}^{\infty} (\beta\xi_w)^s E_{h,t} \left[\widetilde{mrs}_{h,t+s} + \chi\theta_w \hat{w}_{t+s} + (1 + \chi\theta_w) \left(\sum_{j=0}^s \pi_{t+j} - \iota_w \sum_{j=1}^s \pi_{t+j-1} \right) + \tilde{\mu}_{t+s}^w \right].$$

Following the same quasi-differencing as for the optimal price, the recursive form is

$$\hat{w}_{h,t}^* = \frac{(1 - \beta\xi_w)}{1 + \chi\theta_w} E_{h,t} [\widetilde{mrs}_{h,t} + \chi\theta_w \hat{w}_t + \tilde{\mu}_t^w] + (1 - \iota_w \beta\xi_w) E_{h,t} [\pi_t] + \beta\xi_w E_{h,t} [\hat{w}_{h,t+1}^*]. \quad (\text{I.10})$$

Integrating over h , using $\int_0^1 c_{h,t} dh = c_t$ and $\int_0^1 c_{h,t-1} dh = c_{t-1}$,

$$\begin{aligned} \hat{w}_t^* &= \frac{(1 - \beta\xi_w)}{1 + \chi\theta_w} \left[\frac{1}{1 - \varphi/\gamma} (c_t - \varphi/\gamma c_{t-1}) + \chi \bar{E}_t[l_t] + \chi\theta_w \bar{E}_t[\hat{w}_t] + \bar{E}_t[\tilde{\mu}_t^w] \right] \\ &\quad + (1 - \iota_w \beta\xi_w) \bar{E}_t[\pi_t] + \beta\xi_w \int_0^1 E_{h,t} [\hat{w}_{h,t+1}^*] dh. \end{aligned} \quad (\text{I.11})$$

The only difference relative to the complete-markets derivation is that the consumption component $\frac{1}{1-\varphi/\gamma}(c_t - \varphi/\gamma c_{t-1})$ enters as a known aggregate (channel G_1), rather than through $\bar{E}_t[mrs_t]$ (channel G_2). The labor and markup components still require forecasting because they involve aggregates outside the household's information set at the wage-setting stage.

Substituting (I.11) into the wage index (I.7) gives wage dynamics under heterogeneous information,

$$\begin{aligned} \hat{w}_t + \pi_t = & \xi_w(\iota_w \pi_{t-1} + \hat{w}_{t-1}) + \kappa_w \xi_w \left[\frac{1}{1-\varphi/\gamma}(c_t - \varphi/\gamma c_{t-1}) + \chi \bar{E}_t[l_t] + \bar{E}_t[\tilde{\mu}_t^w] \right] \\ & + \psi_w \bar{E}_t[\hat{w}_t] + \psi_{w\pi} \bar{E}_t[\pi_t] + (1 - \xi_w) \beta \xi_w \int_0^1 E_{h,t}[\hat{w}_{h,t+1}^*] dh, \end{aligned} \quad (\text{I.12})$$

where $\kappa_w = \frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w(1+\chi\theta_w)}$, $\psi_{w\pi} \equiv (1 - \xi_w)(1 - \iota_w \beta \xi_w)$ and $\psi_w \equiv \kappa_w \xi_w \chi \theta_w = (1 - \xi_w)(1 - \beta \xi_w) \frac{\chi \theta_w}{1 + \chi \theta_w}$. The real wage depends on its own average expectation with coefficient ψ_w , the wage counterpart of ψ_p . Iterating (I.12) on its own average expectation, as done for inflation in equation (28) of the main text, makes the whole hierarchy of beliefs explicit, with the dependence on order k decaying geometrically at rate ψ_w .

J Impulse response functions

In this section, the remaining impulse responses for the FI and DI models estimated with the dataset including expectations data are displayed in Figures J1-J5.

Figure J6 repeats, for macroeconomic variables, the decomposition of impulse responses into first- and higher-order uncertainty reported for revision variables in the main text. The DI responses (solid black) and the counterfactual without higher-order uncertainty (dashed gray) are essentially indistinguishable. Virtually all of the attenuation and delay that information frictions impart to macroeconomic dynamics comes from first-order learning.

Figure J1: Impulse responses to preference shock

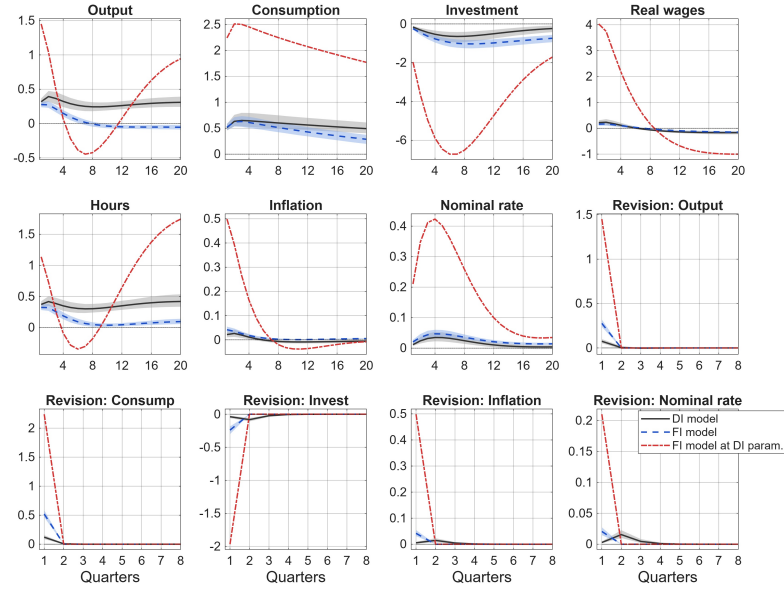


Figure J2: Impulse responses to TFP shock

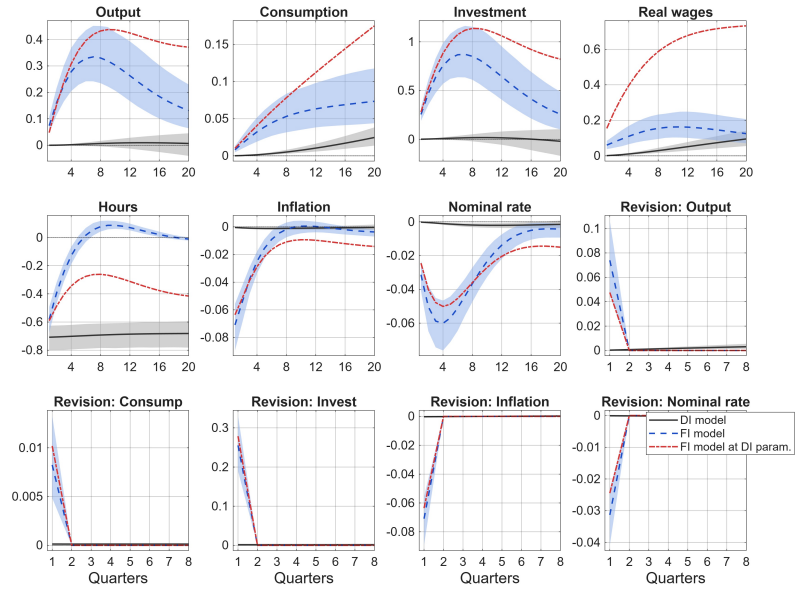


Figure J3: Impulse responses to wage mark-up shock

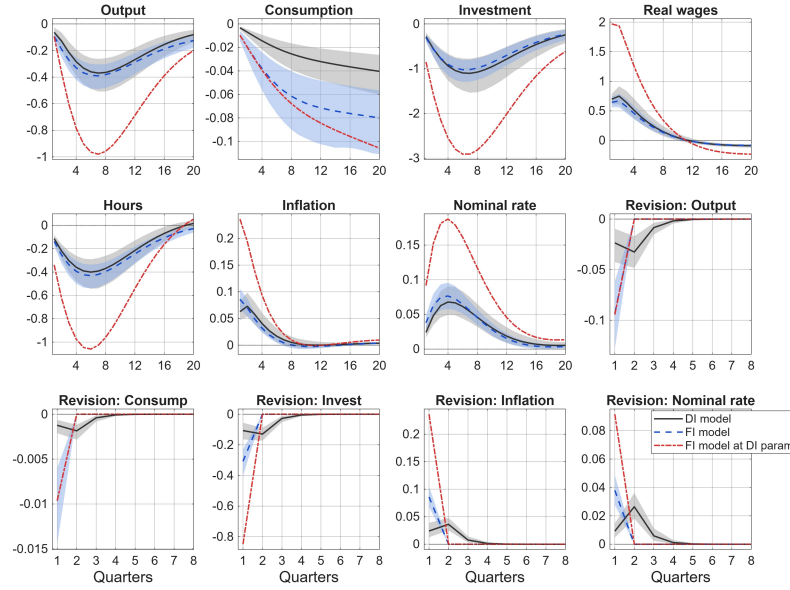


Figure J4: Impulse responses to government expenditure shock

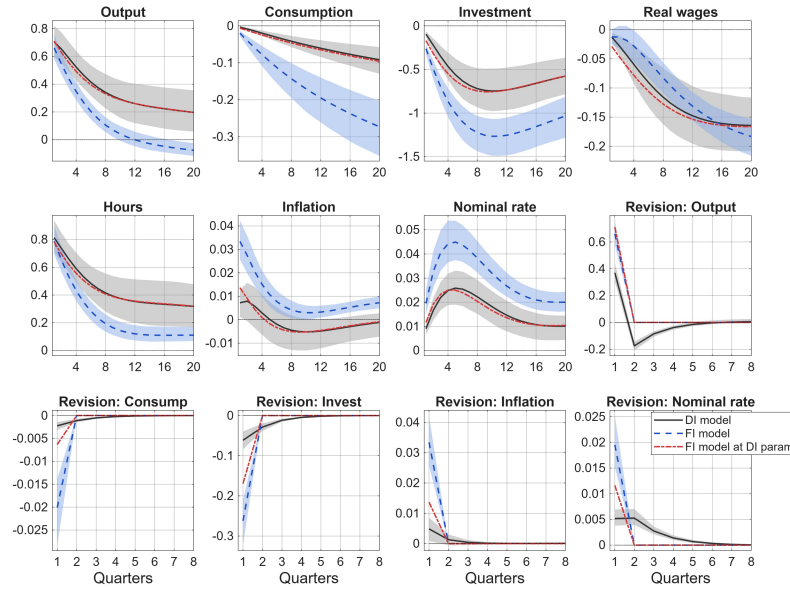


Figure J5: Impulse responses to monetary policy shock

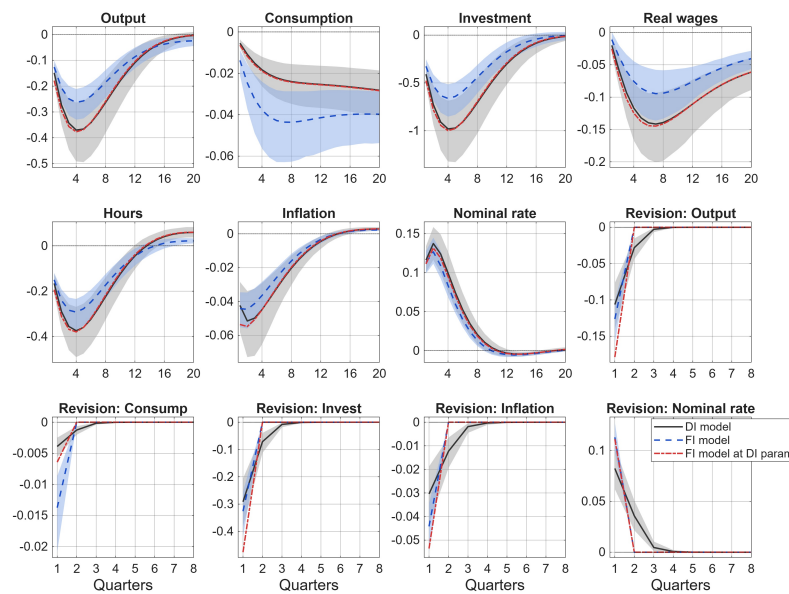
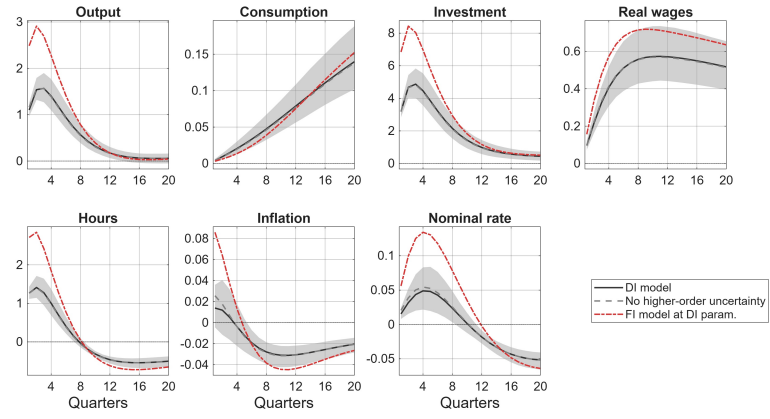
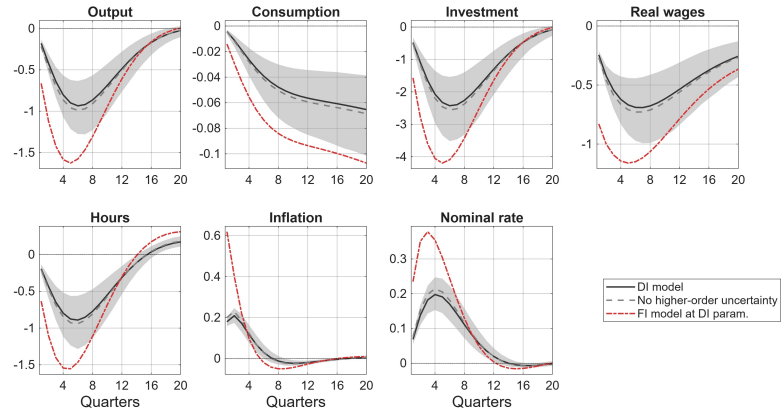


Figure J6: Decomposition of impulse responses



(a) Investment-specific technology shock



(b) Cost-push shock